Betatron tune Measurement

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FFA school, Sep. 8-9, Osaka, 2018

CONTENTS

Betatron oscillation and tune

How to measure tunes

KURNS FFAG, Diagnostics

BETATRON OSCILLATION AND TUNE

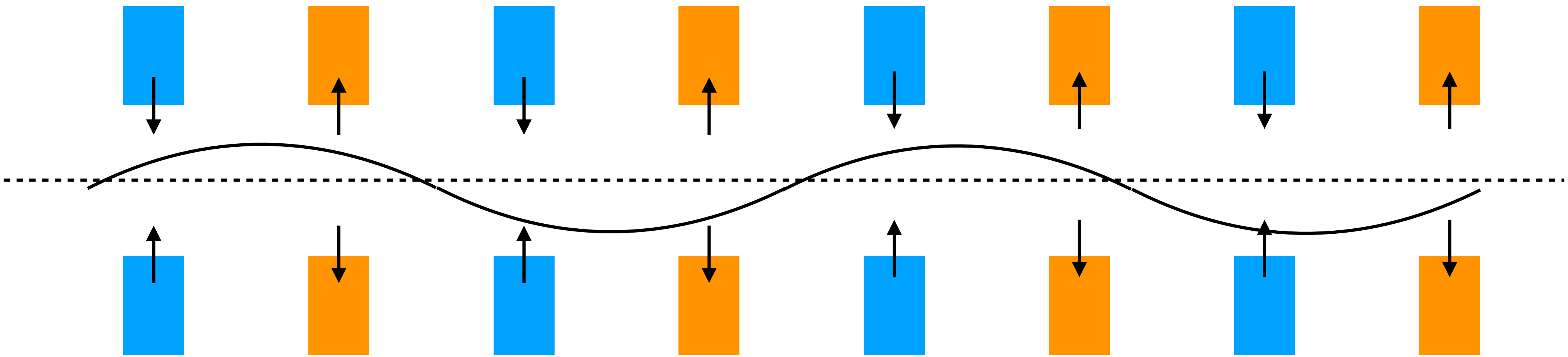
You know what is betatron oscillation

A particle in a circular accelerator affects focusing forces by gradient field,

and oscillates in transverse directions at closed orbit. = *betatron oscillation*

Its frequency depends on field gradient.

Horizontal (f_x) and vertical (f_y) frequencies are different, in general.



Betatron tunes

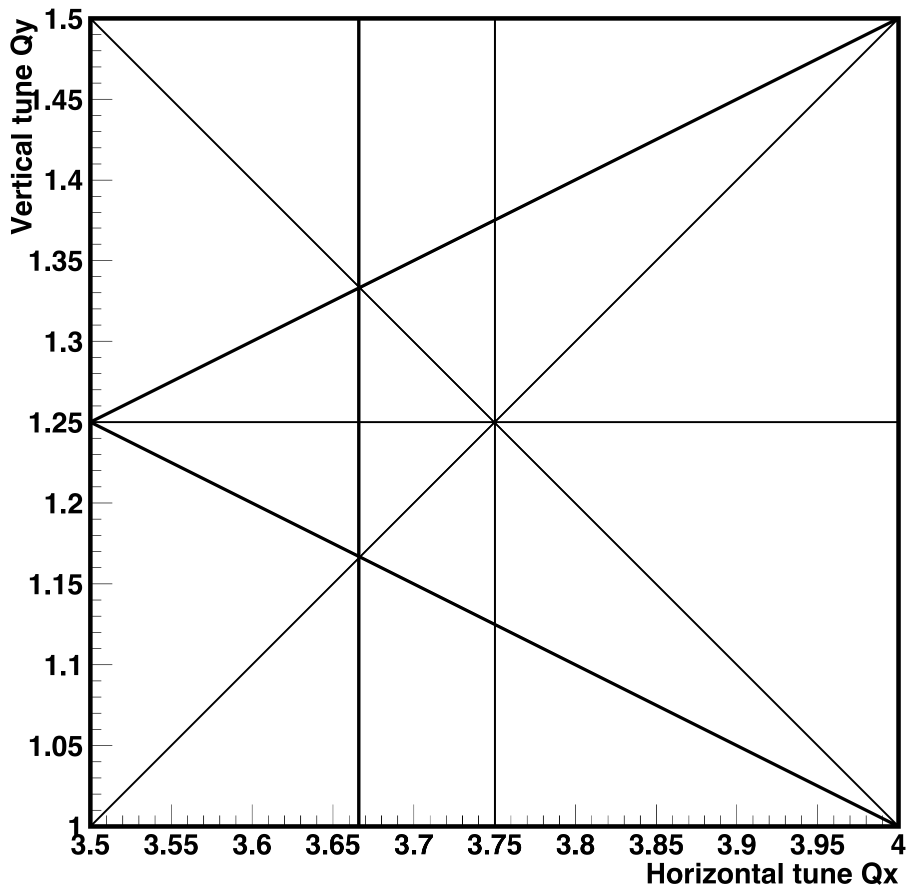
are the frequencies of the betatron oscillations,
divided by the revolution frequency.

$$(Q_x , Q_y) = \left(\frac{f_x}{f_{rev}} , \frac{f_y}{f_{rev}} \right)$$

It is very important parameters
in a circular accelerator.

Beams tend to be lost if the tunes
satisfy the *resonance condition*;

$$\ell Q_x + m Q_y = n \qquad (\ell, m, n = \text{small integers})$$



HOW TO MEASURE TUNES

keywords

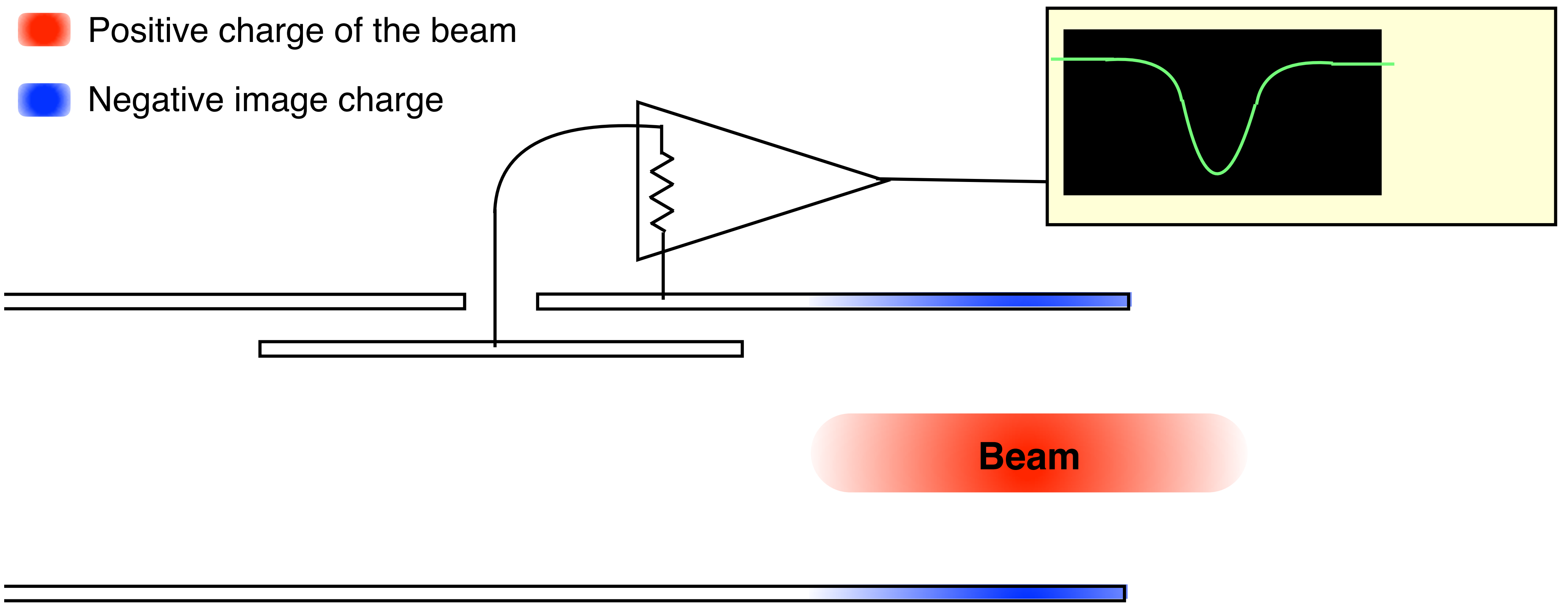
Beam position monitor
Coherent betatron oscillations
Spectrum with betatron sidebands

Electro-static beam position monitor

is composed of a electrode installed in the vacuum chamber.

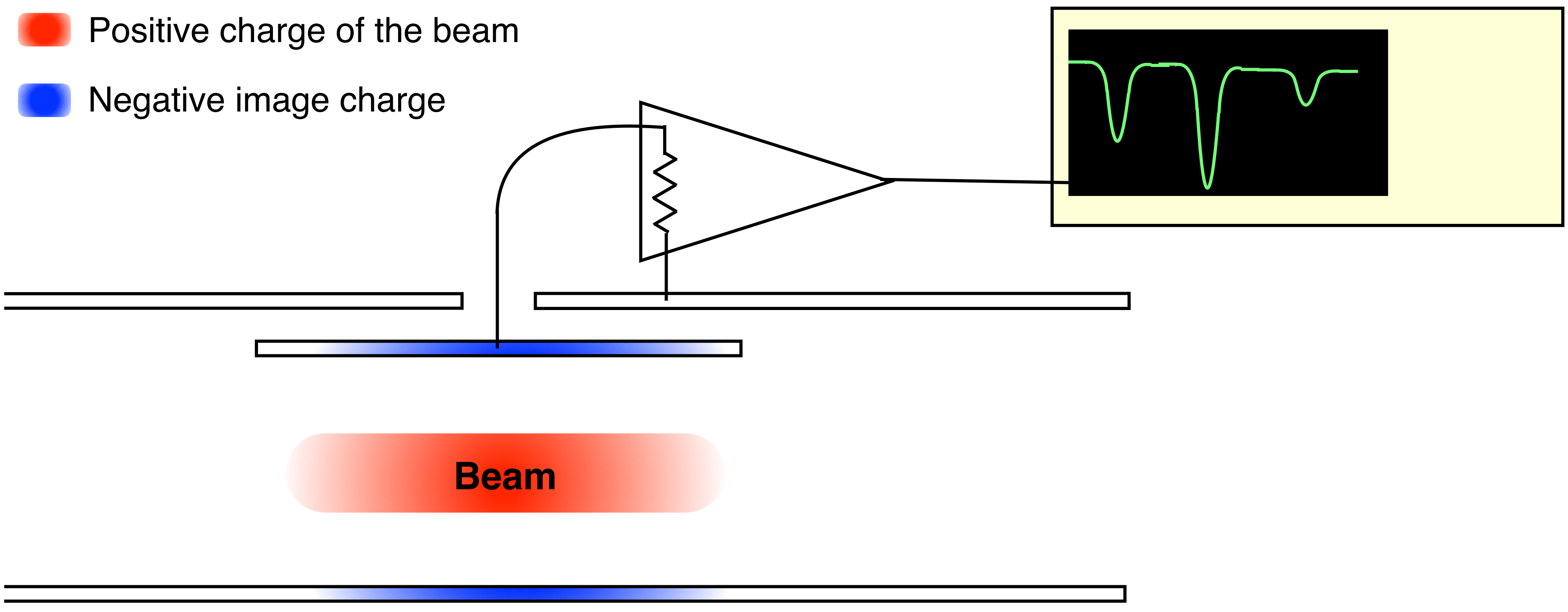
When a charged particle beam pass through,

longitudinal charge distribution can be detected. … Bunch monitor

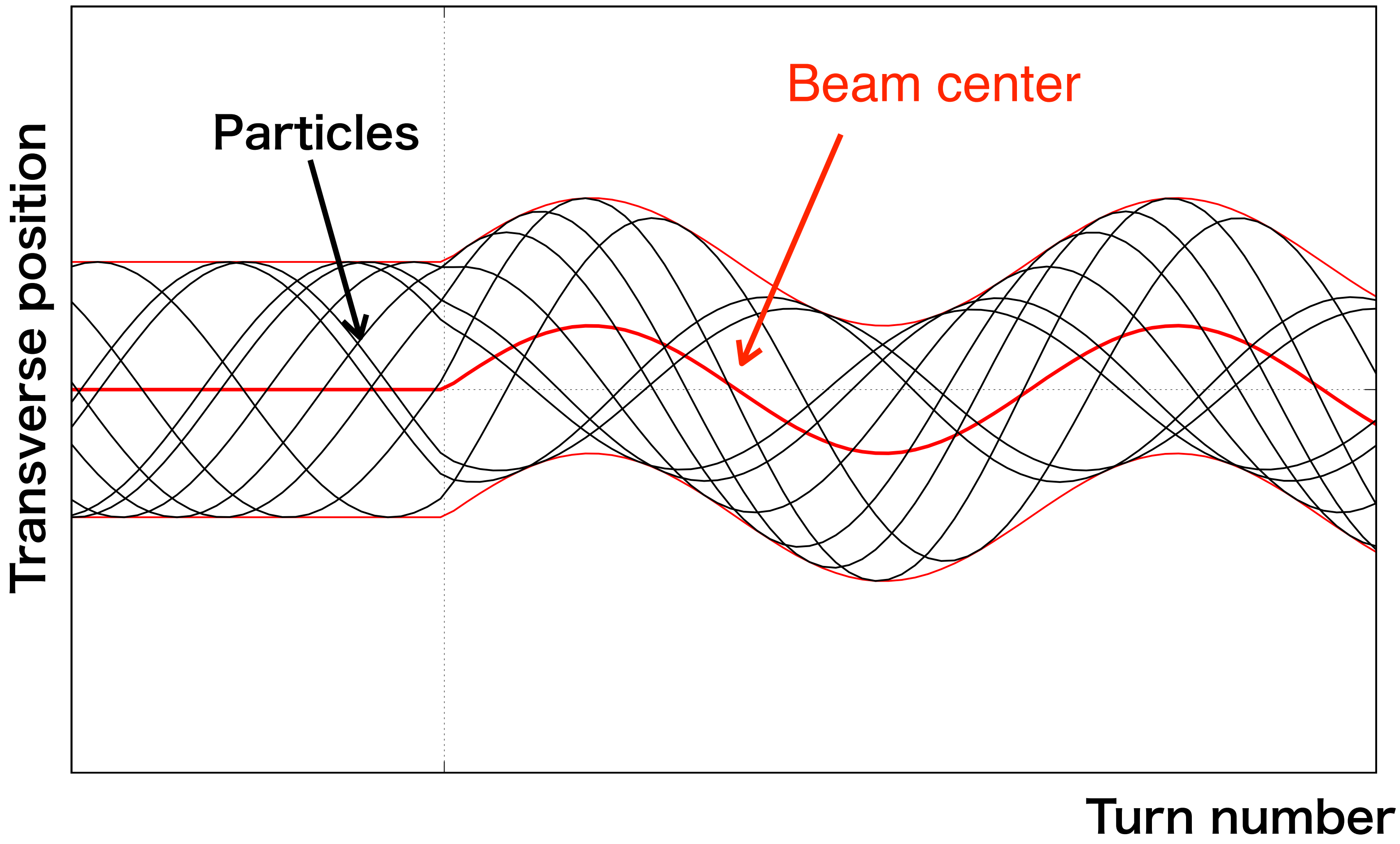


Electro-static beam position monitor

is sensitive to the beam center position.



Coherent oscillations are necessary to measure the frequency by BPM

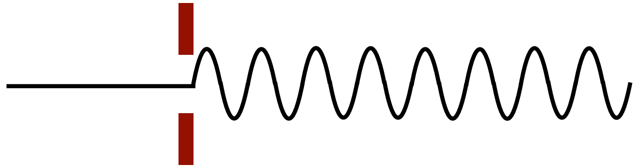


BPM can detect only the position of beam center.

It needs to excite coherent oscillations to observe the betatron oscillation.

How to excite coherent oscillations

(1) Strong pulse kick



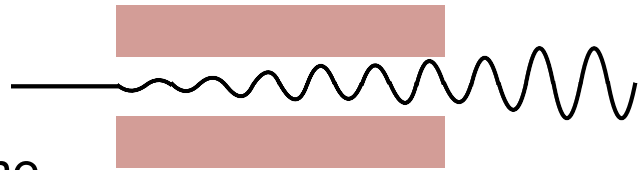
Coherent oscillations can be observed,
if the kick angle is large enough.

$$x_{amplitude} \simeq \beta \, \Delta x' = \beta \, \frac{q \cdot B\ell}{p} = \beta \, \frac{B\ell}{0.48 \; [T \cdot m]} \quad \text{(11 MeV proton)}$$

$$= \beta \, \frac{B\ell}{1.48 \; [T \cdot m]} \quad \text{(100 MeV proton)}$$

In KURNS FFAG,
extraction kicker is available,
only around the extraction orbit.

(2) Rf shaker



Applies transverse field for long duration time.

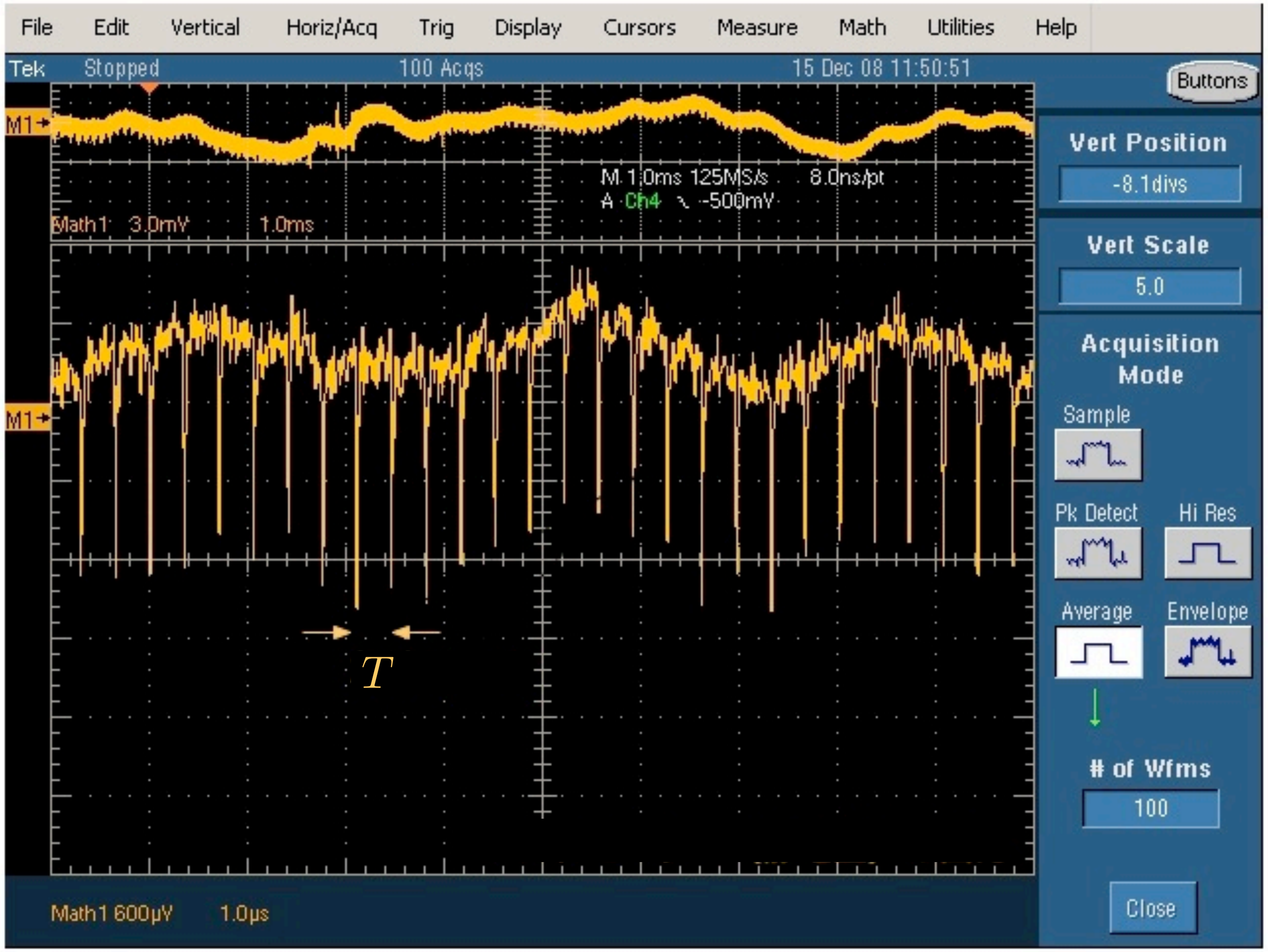
Amplitude of coherent oscillations are
resonantly grow up,

if the shaker frequency is close to the betatron frequency.

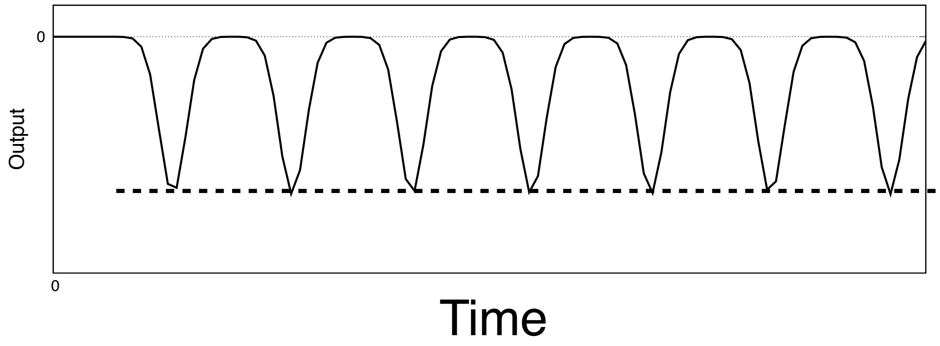
Strong pulse kick → Detect coherent oscillations
Measure the frequency

Rf shaker
tuning its frequency → Detect coherent oscillations
Measure the frequency
→ Beam loss

Signal from Beam Position Monitor



Signal from Beam Position Monitor



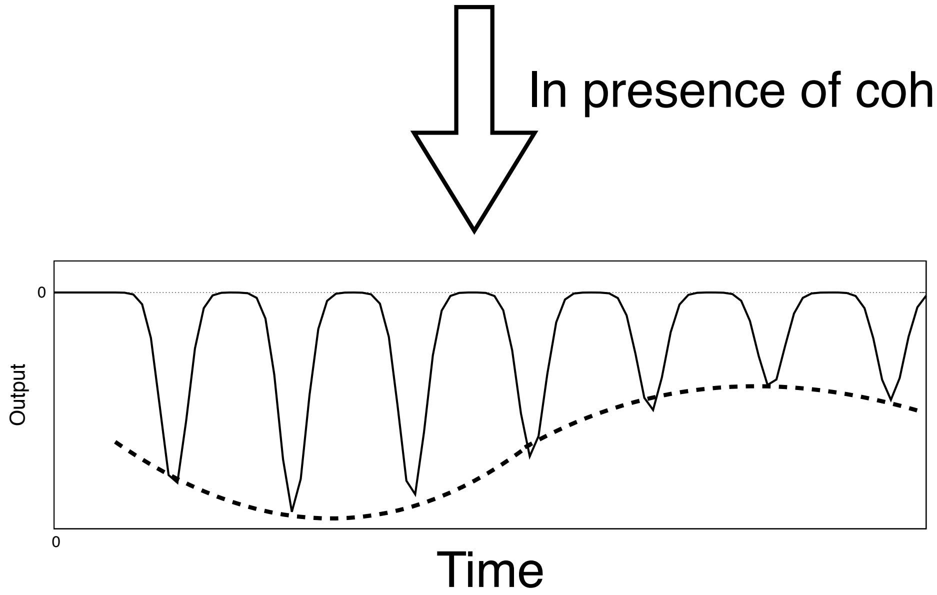
Short bunch approximation

$$V(t) = \sum_{n=0}^{\infty} V_0\,\delta(t - nT_0)$$

T_0 Revolution time

V_0 Proportional to beam intensity

In presence of coherent betatron oscillations



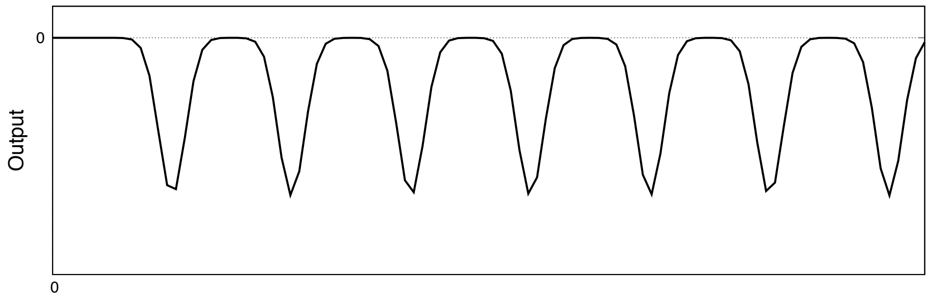
$$V(t) = \sum_{n=0}^{\infty} \underline{(V_0 + \Delta V \cos\omega_\beta t)}\,\delta(t - nT_0)$$

↑ AM factor

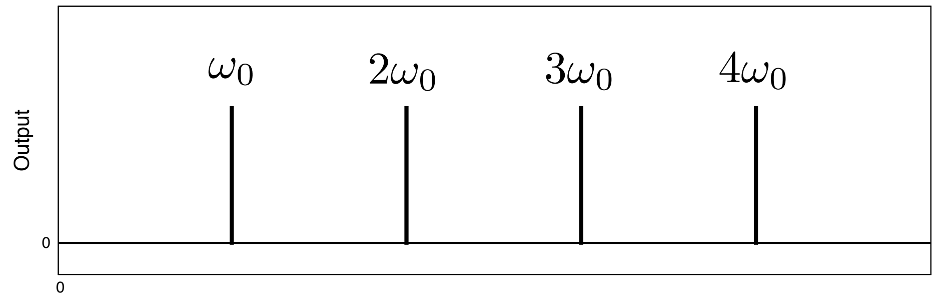
ΔV Betatron amplitude

ω_β Betatron (angular) frequency

Spectrum



Time



Frequency

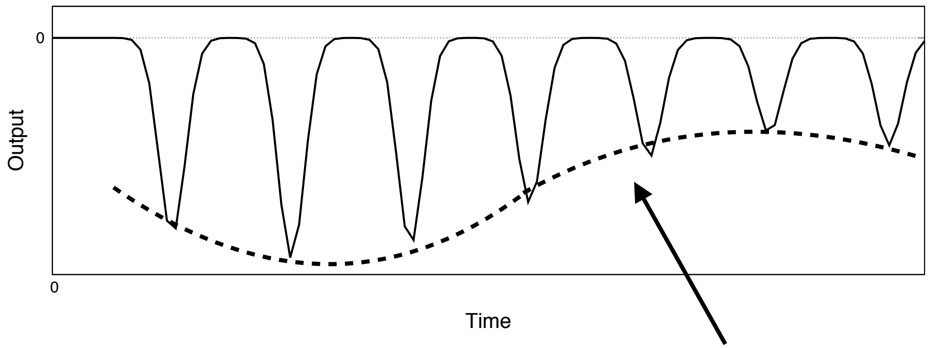
$$V(t) = \sum_{n=0}^{\infty} V_0\, \delta(t - nT_0)$$

T_0 Revolution time

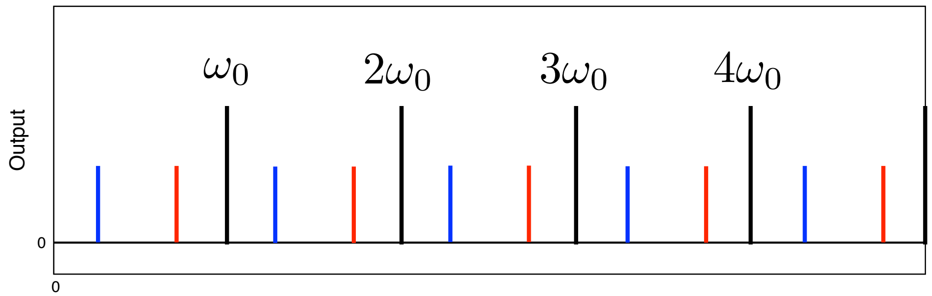
V_0 Proportional to beam intensity

$$\begin{aligned} \tilde{V}(\omega) &= \int \sum_{n=0}^{\infty} V_0\, \delta(t - nT_0) \exp(i\omega t)\, \mathrm{d}t \\ &= V_0 \sum_{m=-\infty}^{\infty} \exp(im\omega T_0) \\ &= \omega_0 V_0 \sum_{n=-\infty}^{\infty} \boxed{\delta(\omega - n\omega_0)} \end{aligned}$$

Spectrum (with betatron oscillation)



Time



Frequency

ΔV Betatron amplitude

ω_β Betatron (angular) frequency

T_0 Revolution time

V_0 Proportional to beam intensity

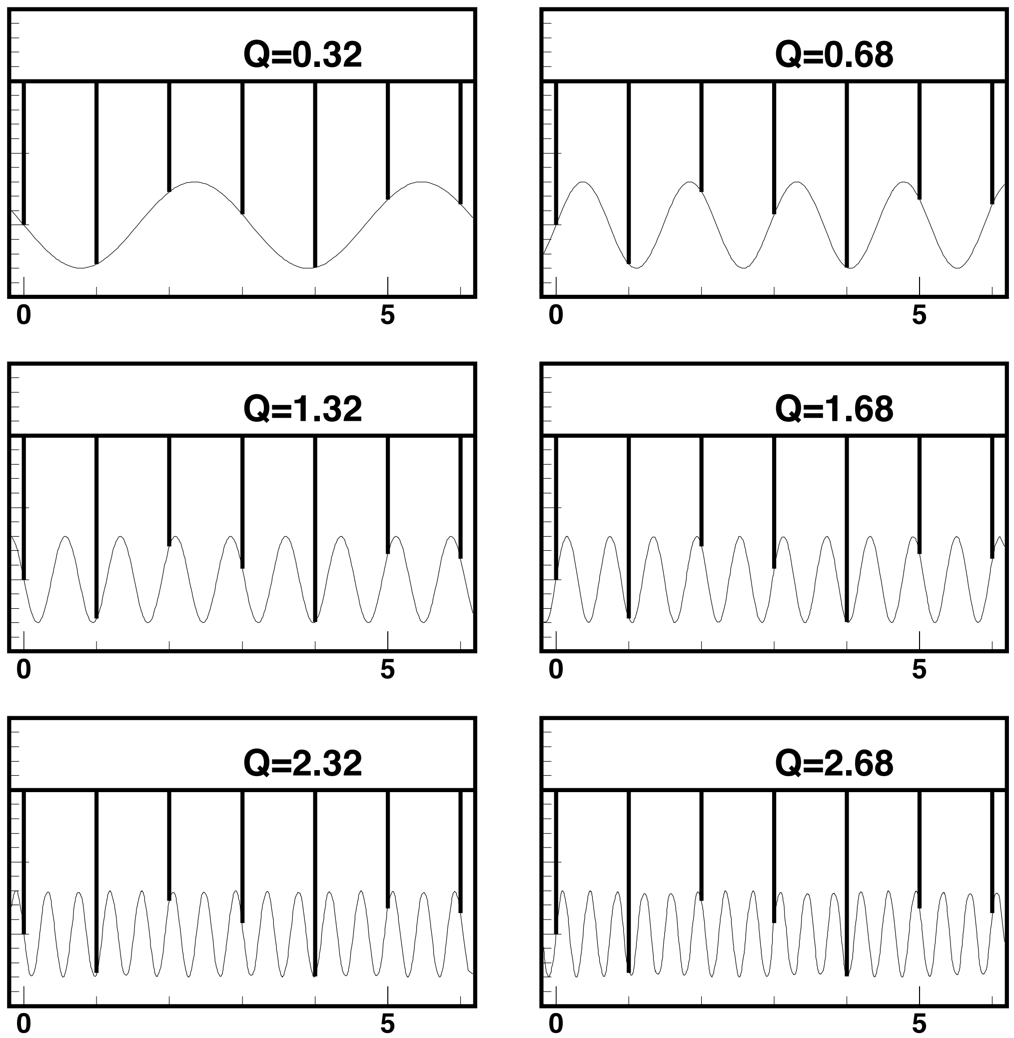
$$\omega_0 = \frac{2\pi}{T_0}$$

$$V(t) = \sum_{n=0}^{\infty} (V_0 + \Delta V \cos \omega_\beta t)\, \delta(t - nT_0)$$

$$\begin{aligned}
\tilde{V}(\omega) &= \int \sum_{n=0}^{\infty} (V_0 + \Delta V \cos \omega_\beta t)\, \delta(t - nT_0) \exp(i\omega t)\, \mathrm{d}t \\
&= \omega_0 V_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) + \frac{\Delta V}{2} \sum_{m=-\infty}^{\infty} \left(e^{im(\omega+\omega_\beta)t} + e^{im(\omega-\omega_\beta)t} \right) \\
&= \omega_0 V_0 \sum_{n=-\infty}^{\infty} \boxed{\delta(\omega - n\omega_0)} + \frac{\omega_0 \Delta V}{2} \sum_{m=-\infty}^{\infty} \left(\boxed{\delta[\omega - (m\omega_0 - \omega_\beta)]} + \boxed{\delta[\omega - (m\omega_0 + \omega_\beta)]} \right)
\end{aligned}$$

Ambiguity of measured tune

$$\omega_0 V_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) + \frac{\omega_0 \Delta V}{2} \sum_{m=-\infty}^{\infty} \Big(\delta[\omega - (m\omega_0 - \omega_\beta)] + \delta[\omega - (m\omega_0 + \omega_\beta)] \Big)$$



$$\omega = \begin{cases} m\omega_0 \\ m\omega_0 + \omega_\beta & = \omega_0(m+Q) \\ m\omega_0 - \omega_\beta & = \omega_0(m-Q) \end{cases}$$

$$m = 0\ ,\ \pm 1\ ,\ \pm 2\ ,\ \pm 3\ ,\ \cdots$$

Signal are the same for these Q values

You must choose one of them, knowing designed tune value.

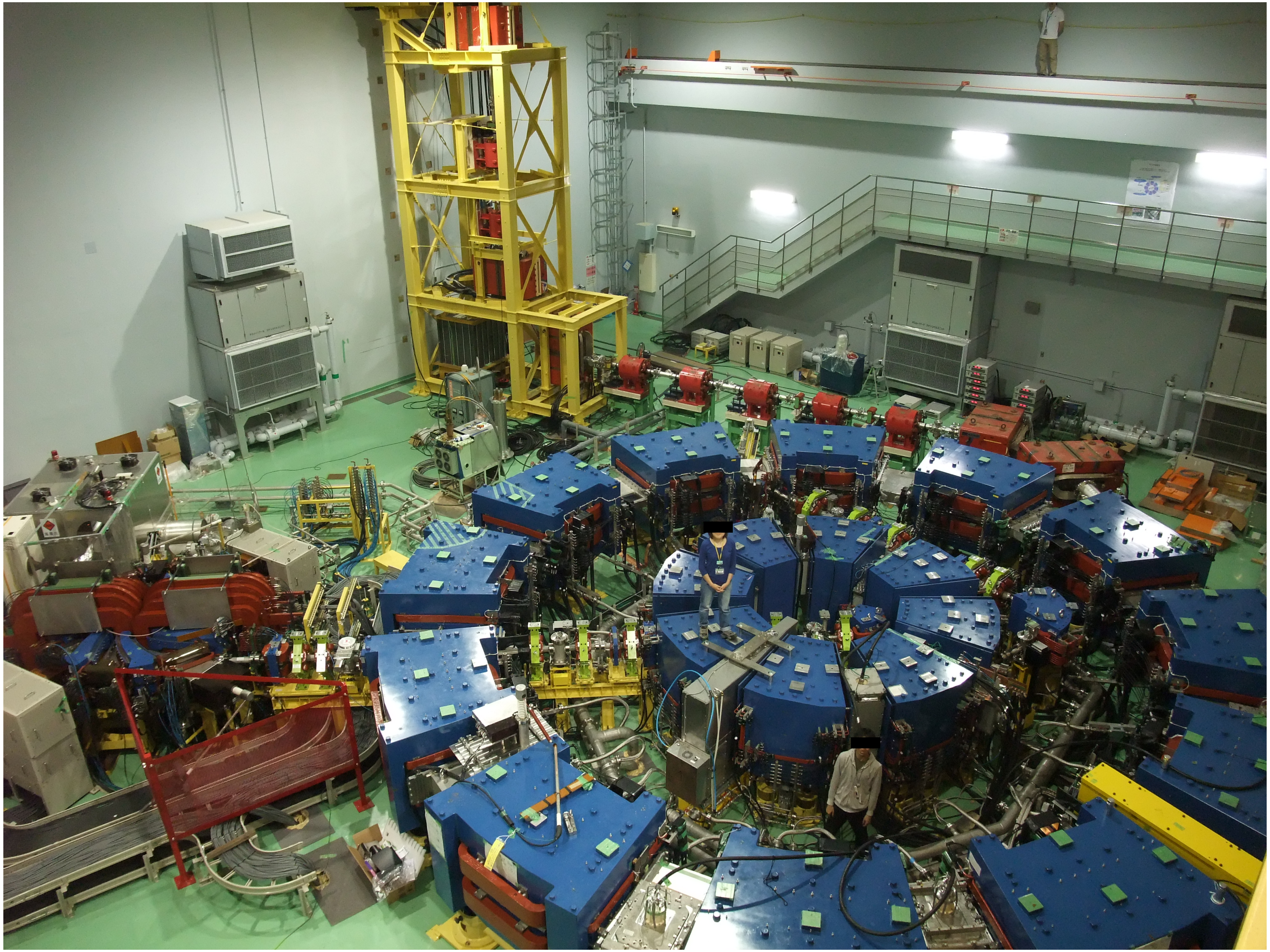
KURNS FFAG,
DIAGNOSTICS

keywords

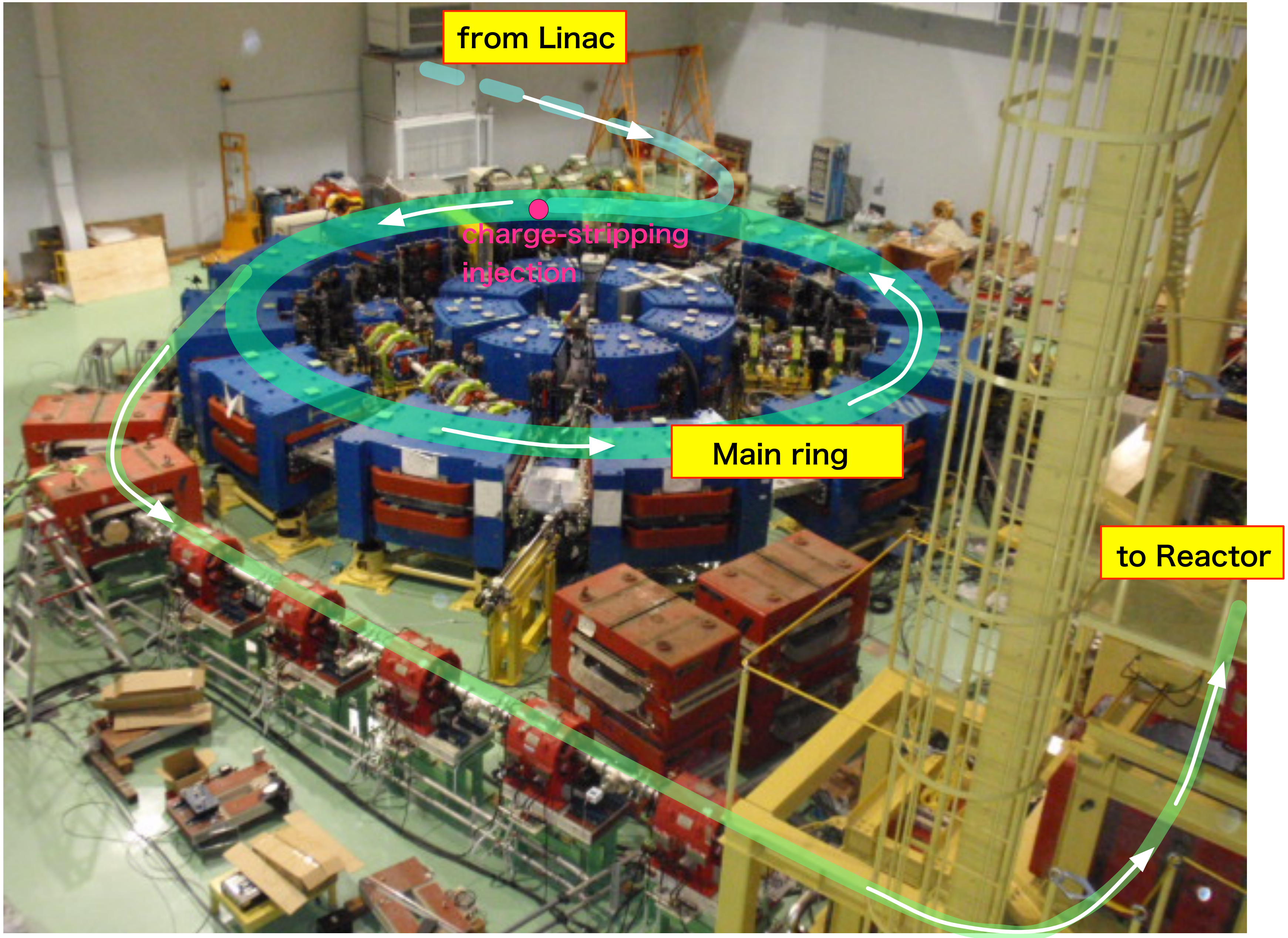
Beam position monitor
Rf shaker

Frequency, Amplitude, Waveform

KURNS FFAG Main Ring



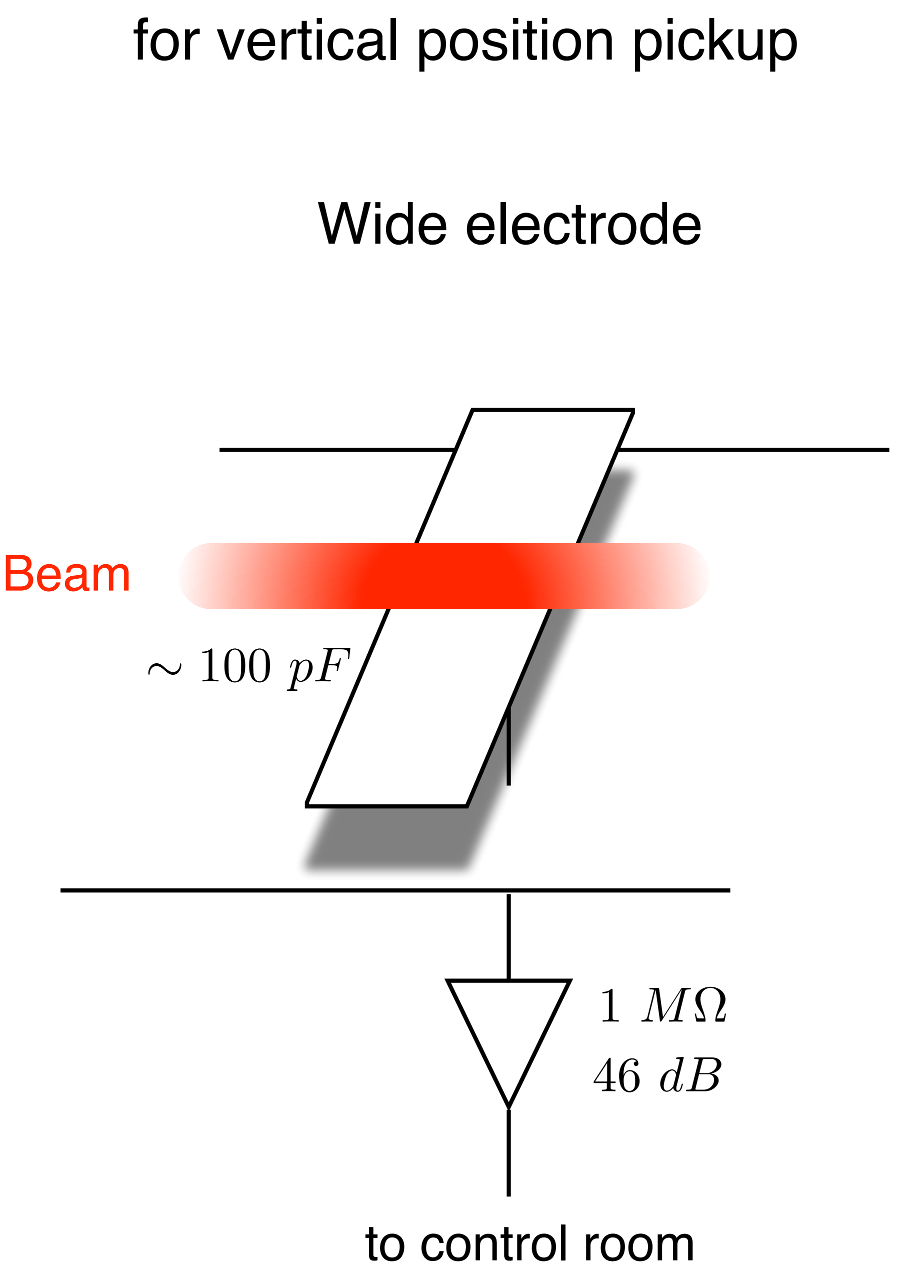
KURNS FFAG Main Ring

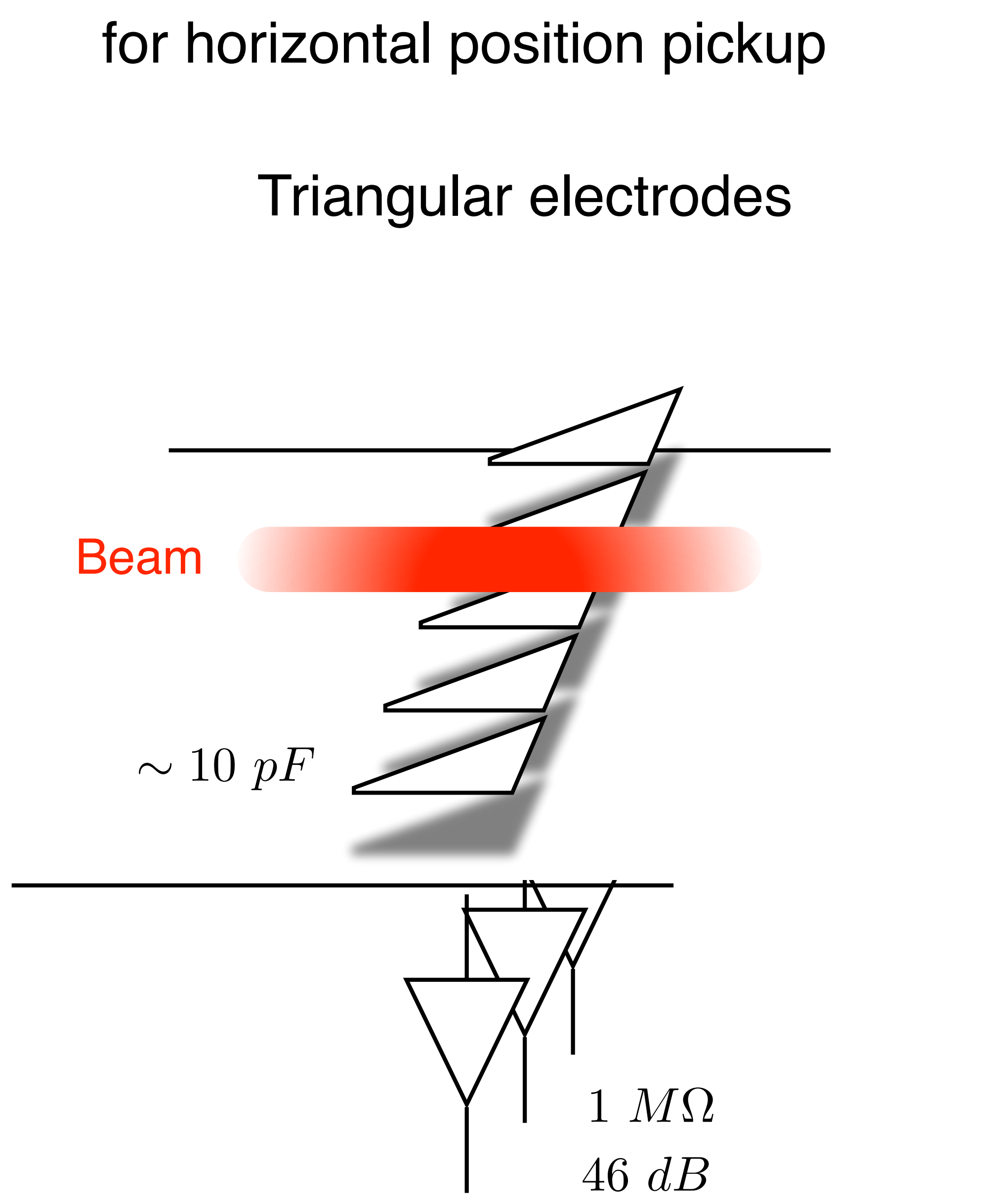


Main Parameters

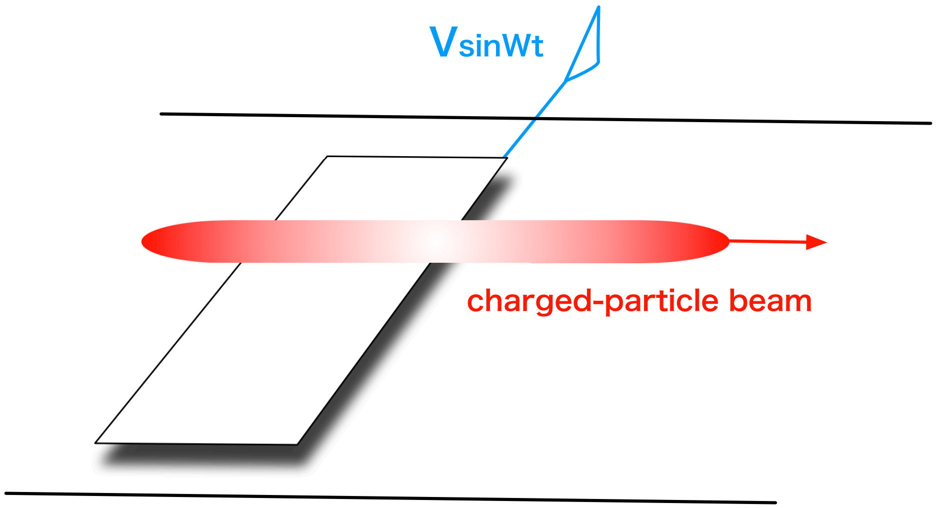
Particle	Proton (H- beams are injected)
Cell	Scaling FFAG, Radial DFD x 12, k=7.6
Orbit radius	4.6 - 5.3 m
Revolution	1. 6 - 4.3 MHz
Designed tune	~(3.7, 1.4)

BPM in KURNS FFAG



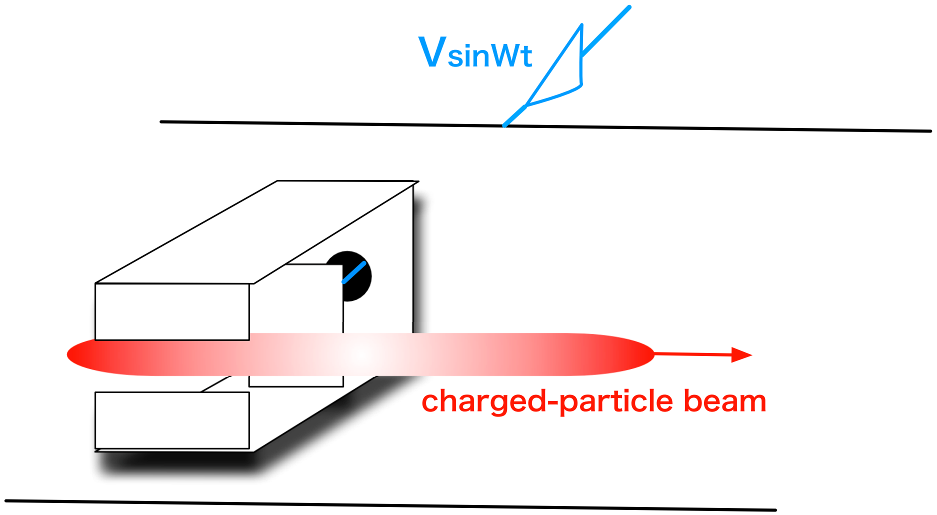


Shaker in KURNS FFAG



for Vertical excitation

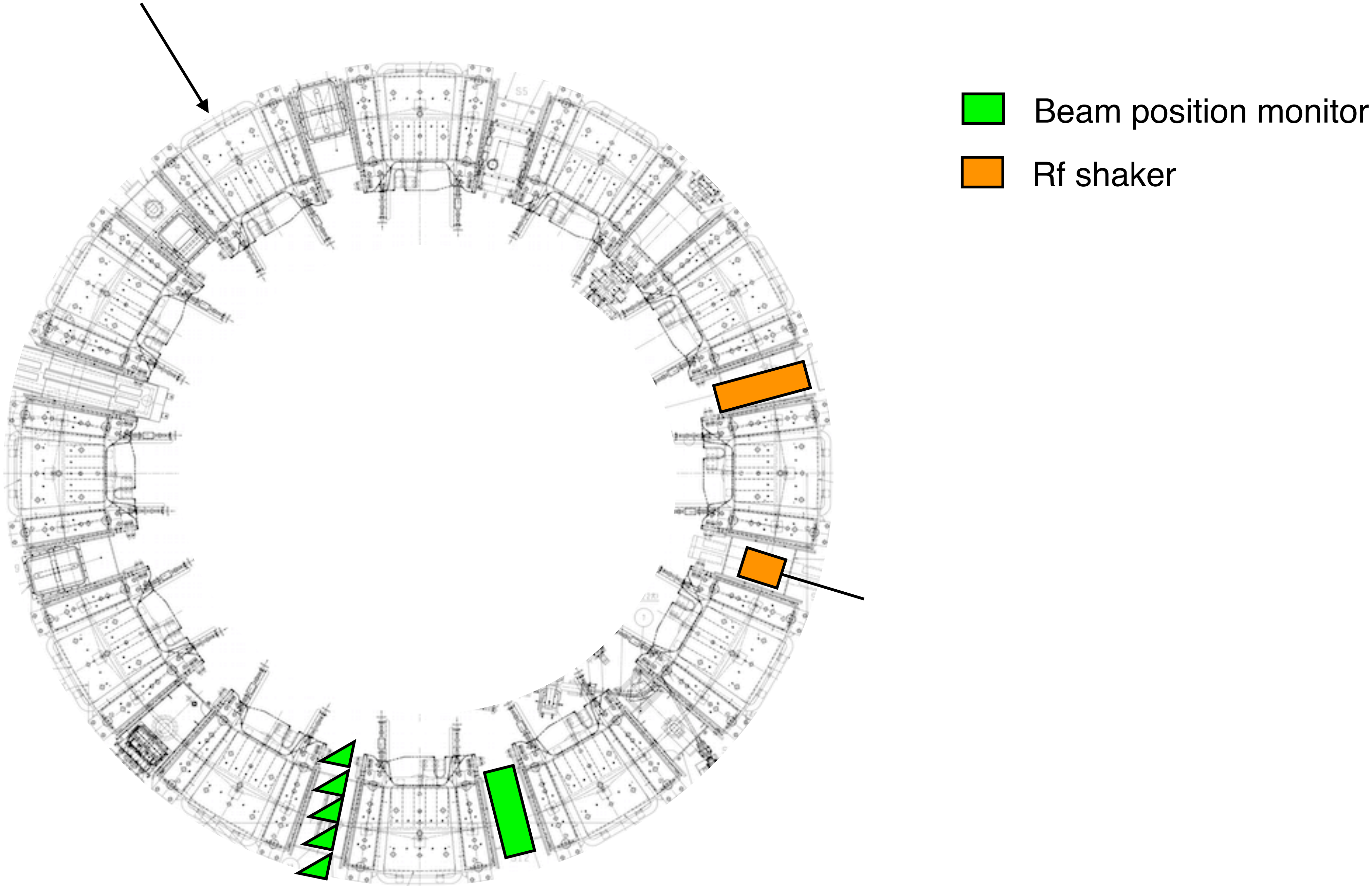
Same as the vertical BPM



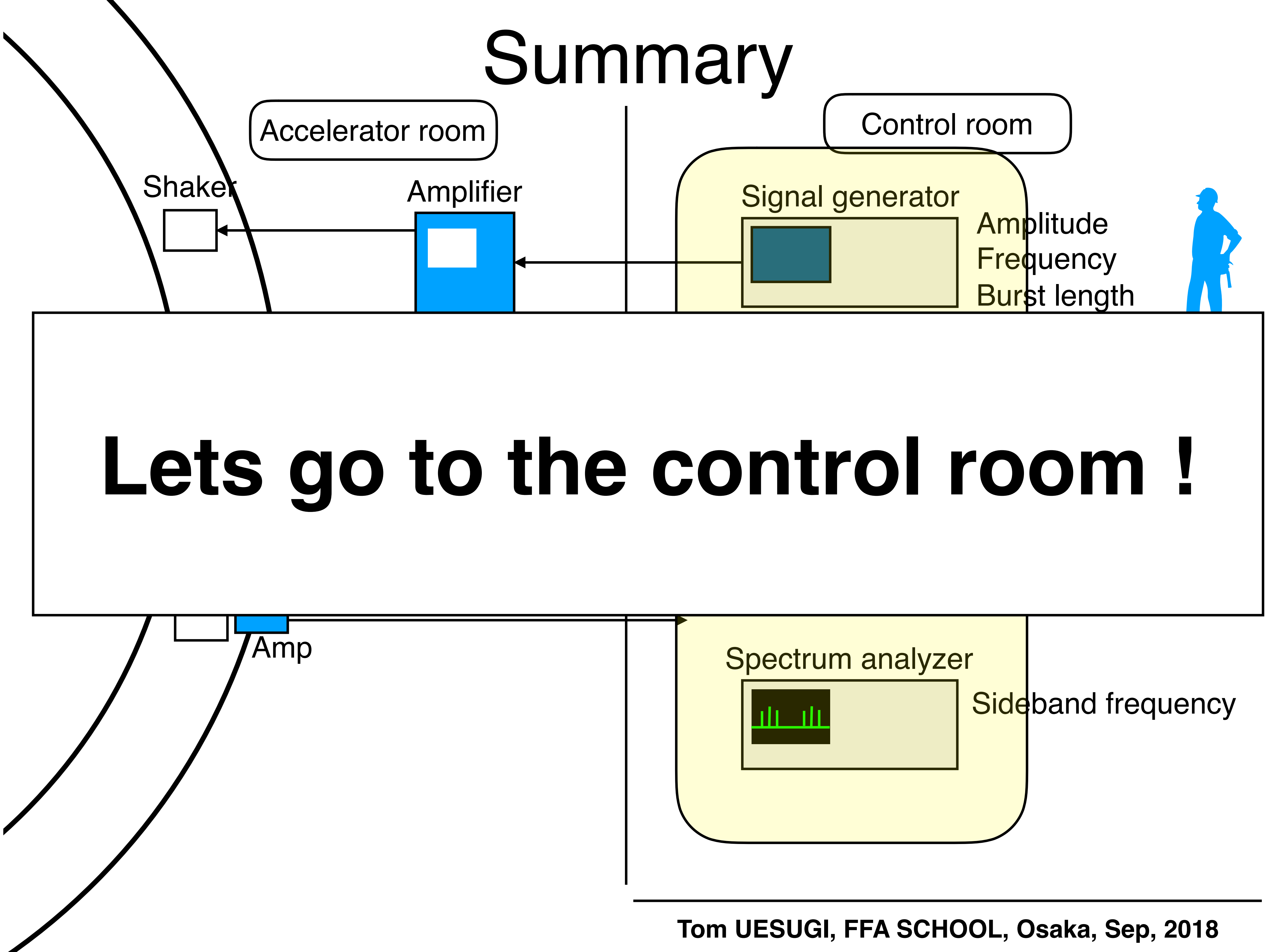
for Horizontal excitation

Remote controlled in horizontal direction

Positions of the monitors and shakers



Summary



Lets go to the control room !