RF acceleration in a FFAG

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KURNS

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CONTENTS

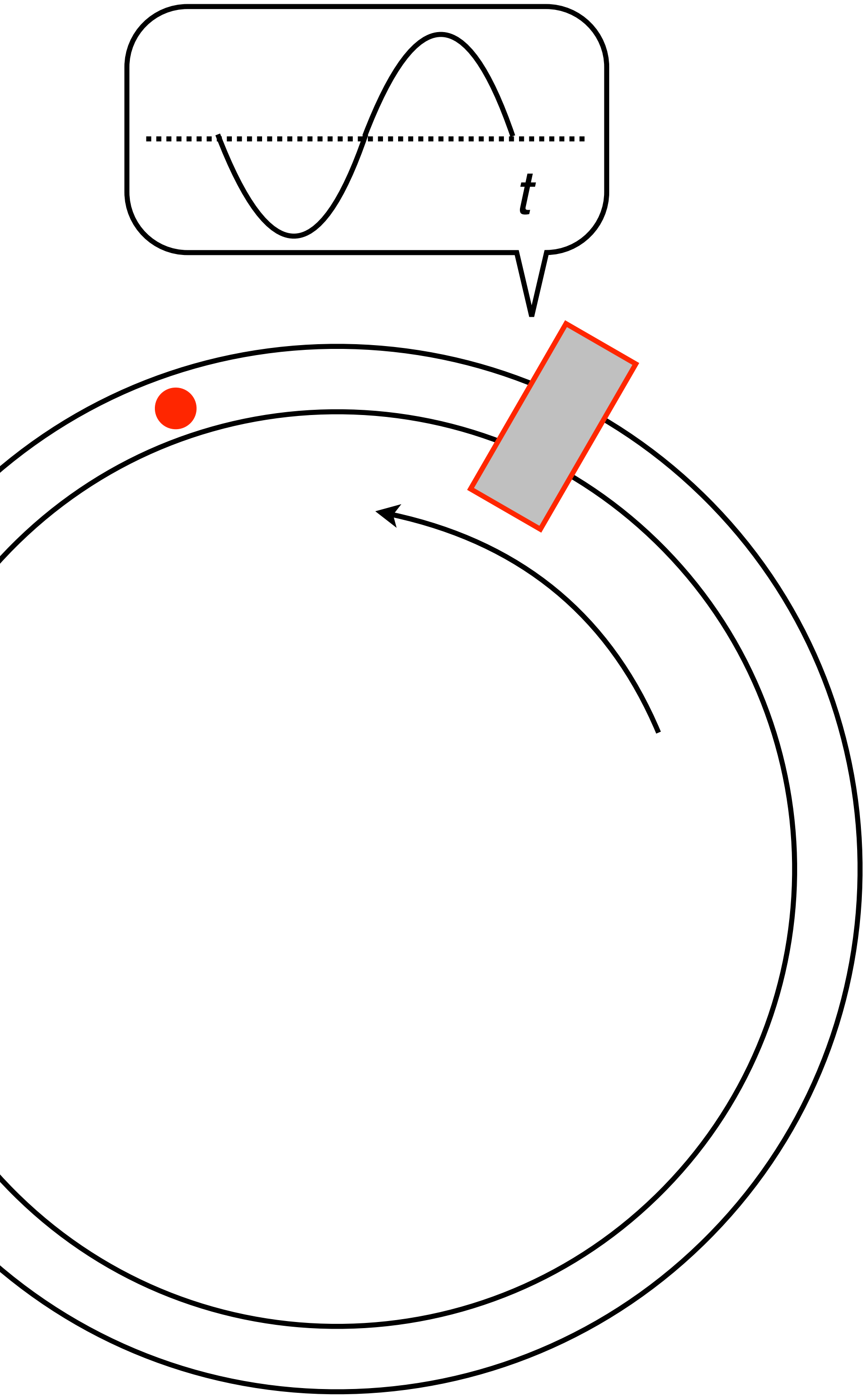
GENERAL THEORY OF RF ACCELERATION

keywords

Momentum compaction, Slippage
Phase stability, Synchrotron oscillation,
Rf bucket

Frequency, Amplitude, Waveform

Rf cavity



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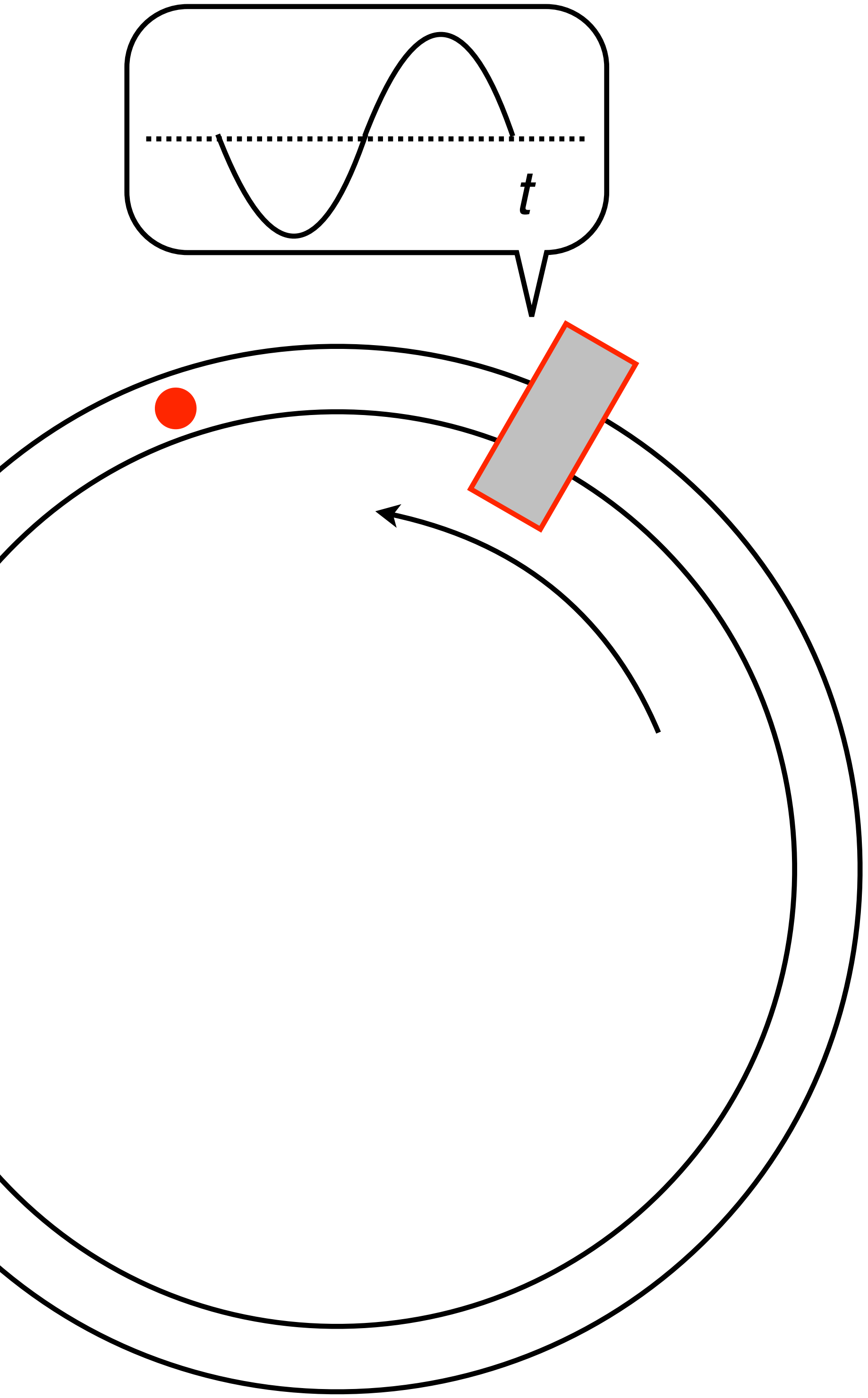
Rf cavity



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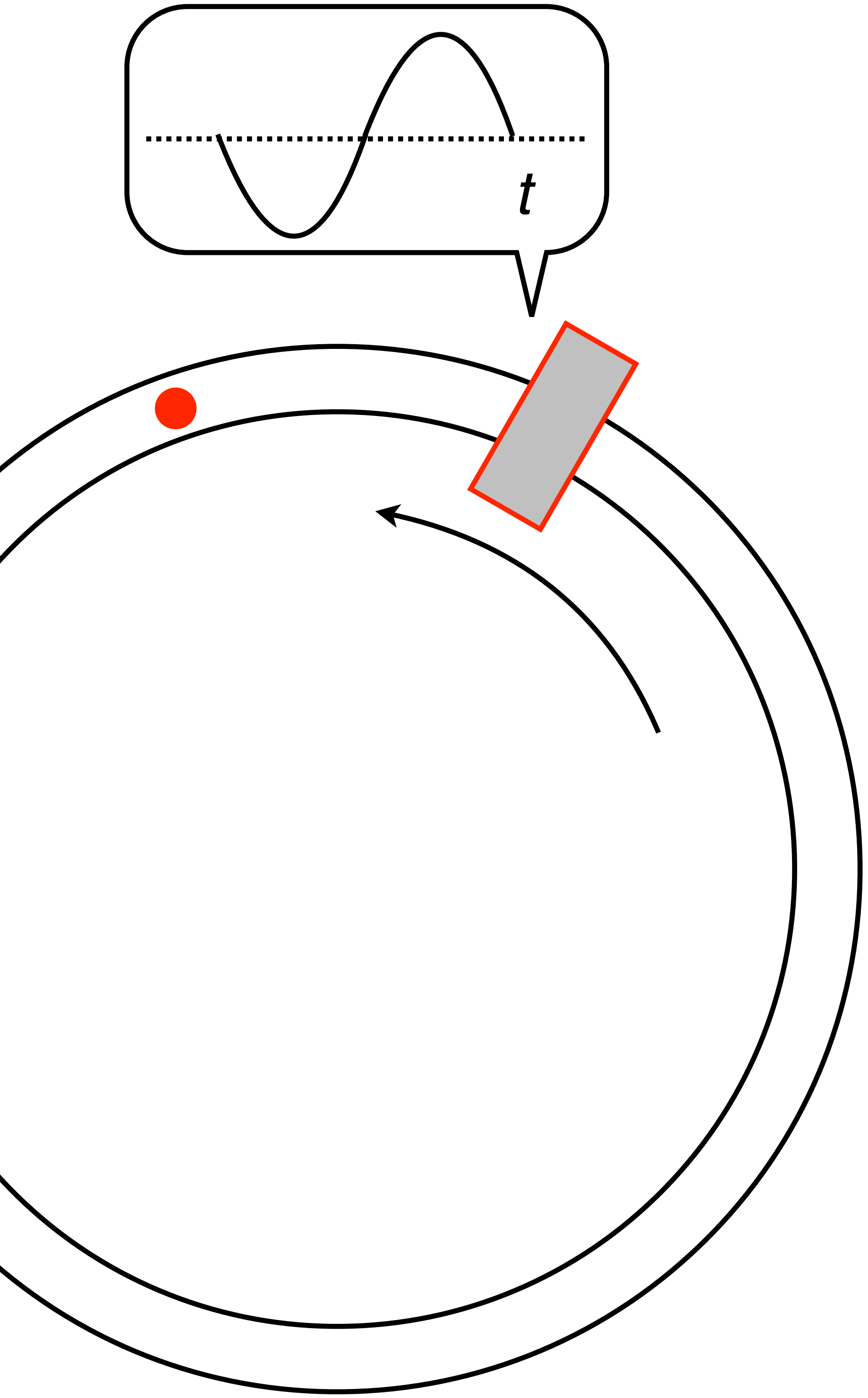
Rf cavity



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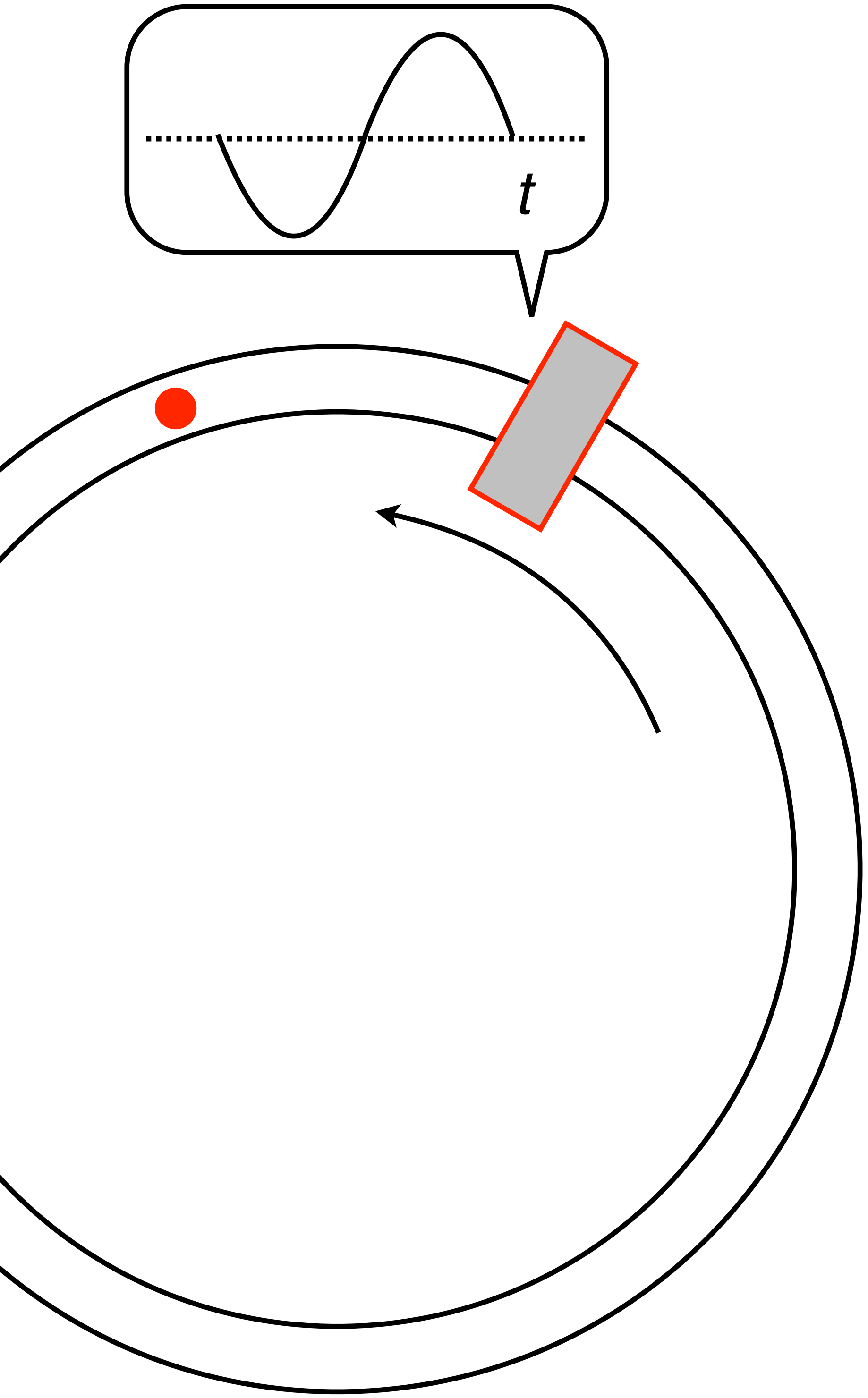

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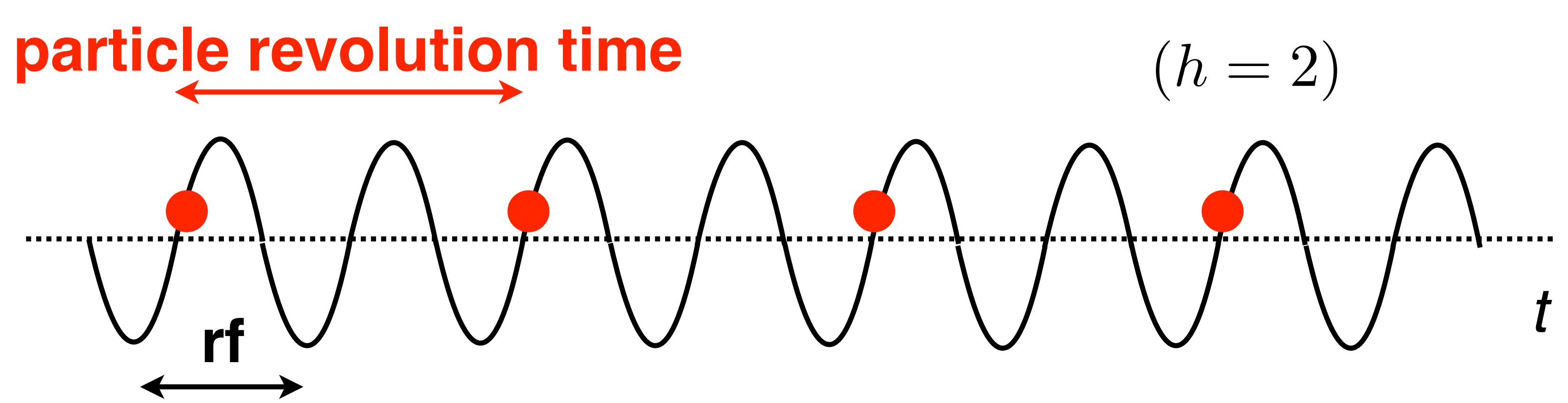

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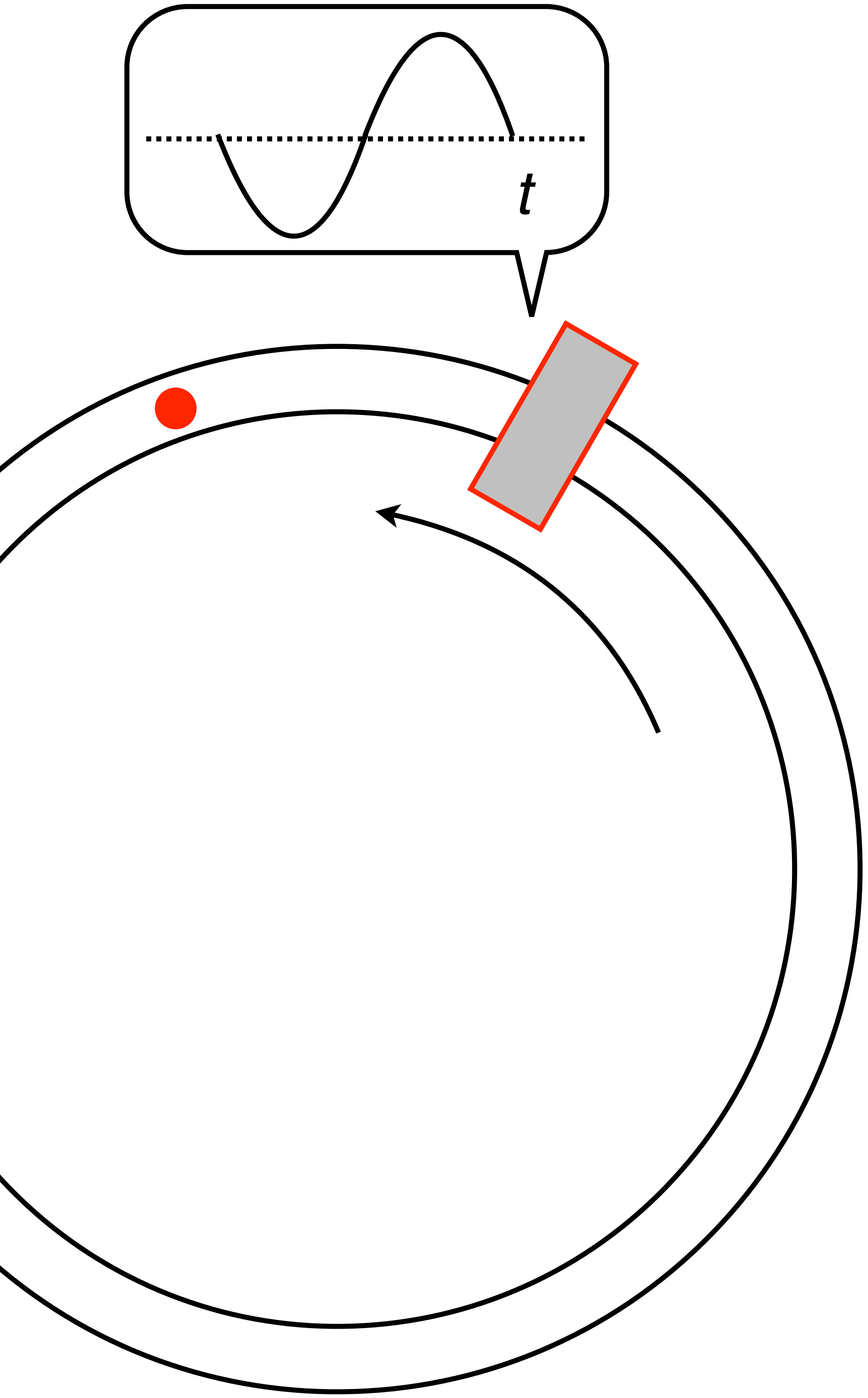

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$$\frac{c}{\pi \times 10\ m} \simeq 10\ MHz$$

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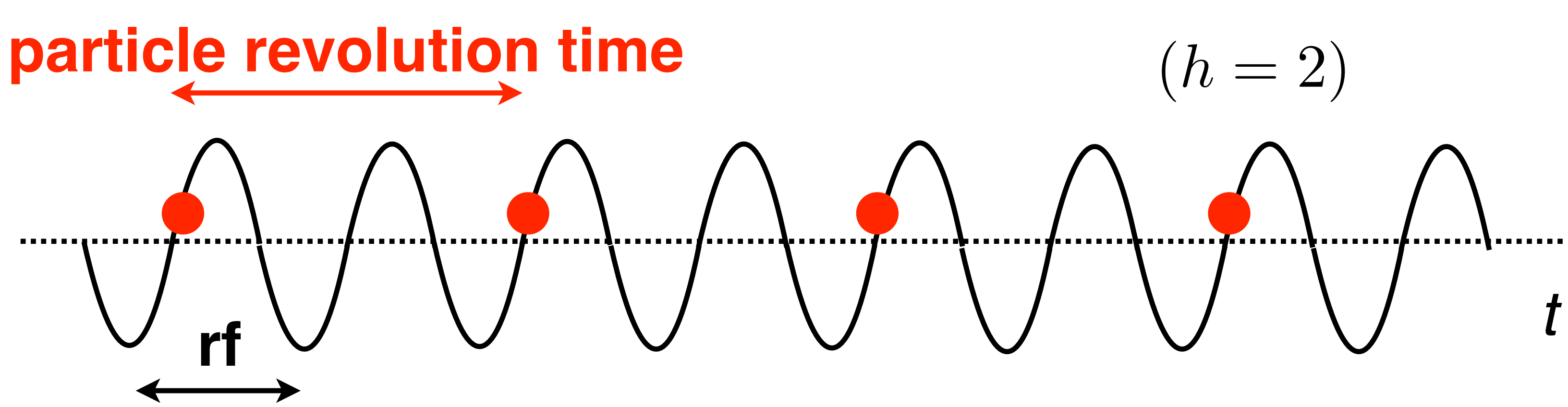

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Frequency — synchronous energy

Amplitude — stability

Particles are accelerated when rf frequency is varied smoothly.

Let us play with a simulation game, later.

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Principle of stable rf acceleration

(1) Revolution time of a particle depends on its energy
(2) Energy gain at rf depends on arrival time

Revolution time of off-momentum particle

$$T = \frac{C}{v}$$

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$$\frac{\mathrm{d}C}{C} \quad = \frac{1}{C}\oint_{ring} D(p)\frac{\mathrm{d}p}{p}\,\mathrm{d}\phi$$

<— function of B

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Higher momentum —> Longer circumference

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$$\frac{\mathrm{d}v}{v} = \mathrm{d}\ln\left(\frac{p}{E}\right) = \left(1 - \frac{p^2}{E^2}\right)\frac{\mathrm{d}p}{p} = \frac{1}{\gamma^2}\frac{\mathrm{d}p}{p}$$ <— kinematics

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<— kinematics

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Higher momentum —> Longer circumference

η : slippage factor

Higher momentum —> Longer revolution time ($\eta > 0$)
Shorter revolution time ($\eta < 0$)

Revolution time in a FFAG

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v : Velocity

depends on momentum and magnetic field.

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Conventional synchrotron

$T(p,t)$ is defined at each instant,
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$$T(p_s + \mathrm{d}p) = T(p_s) + \eta \frac{\mathrm{d}p}{p_s}$$

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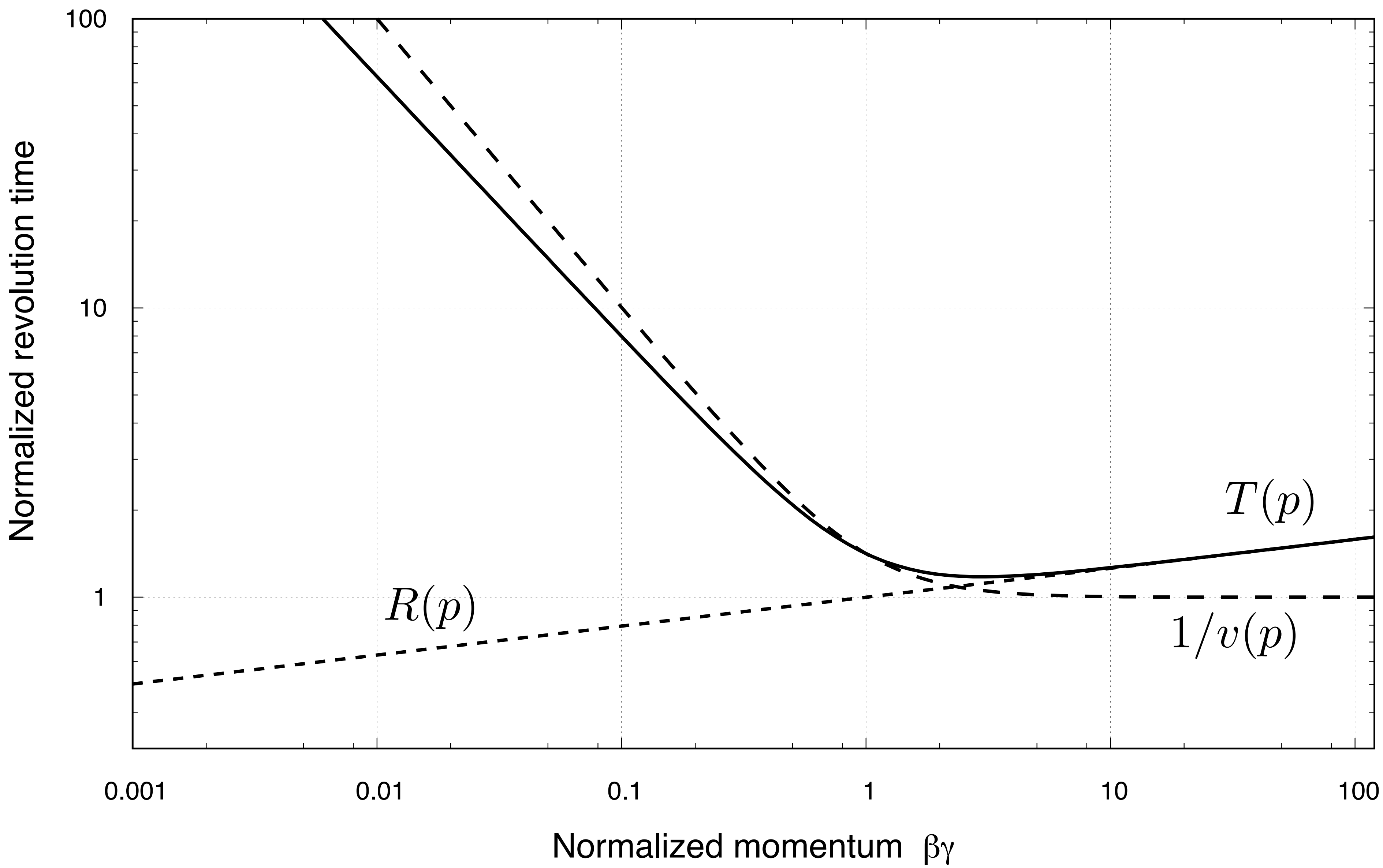
FFAG

Magnetic field is constant.
$T(p)$ can be globally defined independent of time.

Global function T(p) is defined in a FFAG

$$T = \frac{C}{v}$$



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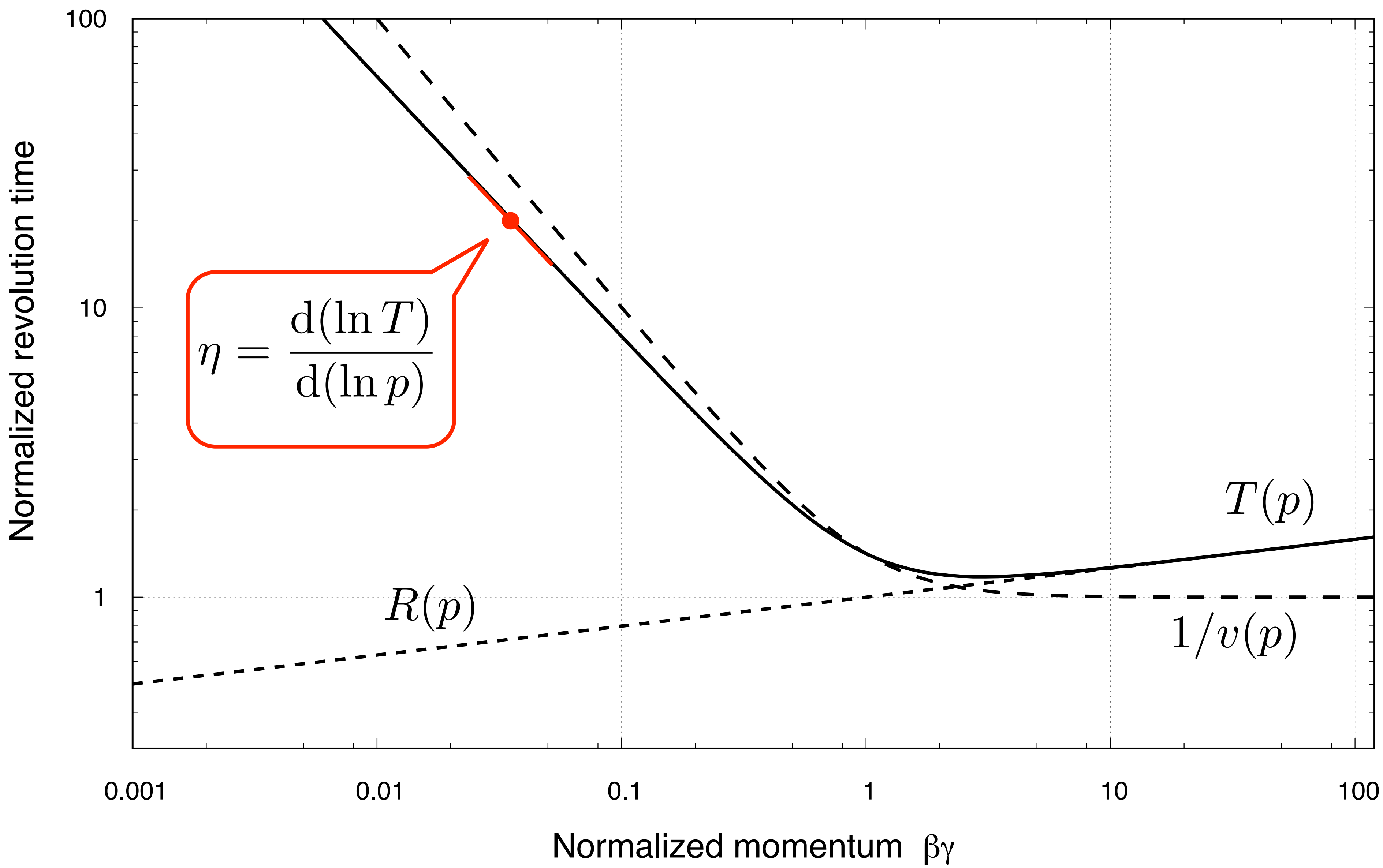

Global function T(p) is defined in a FFAG

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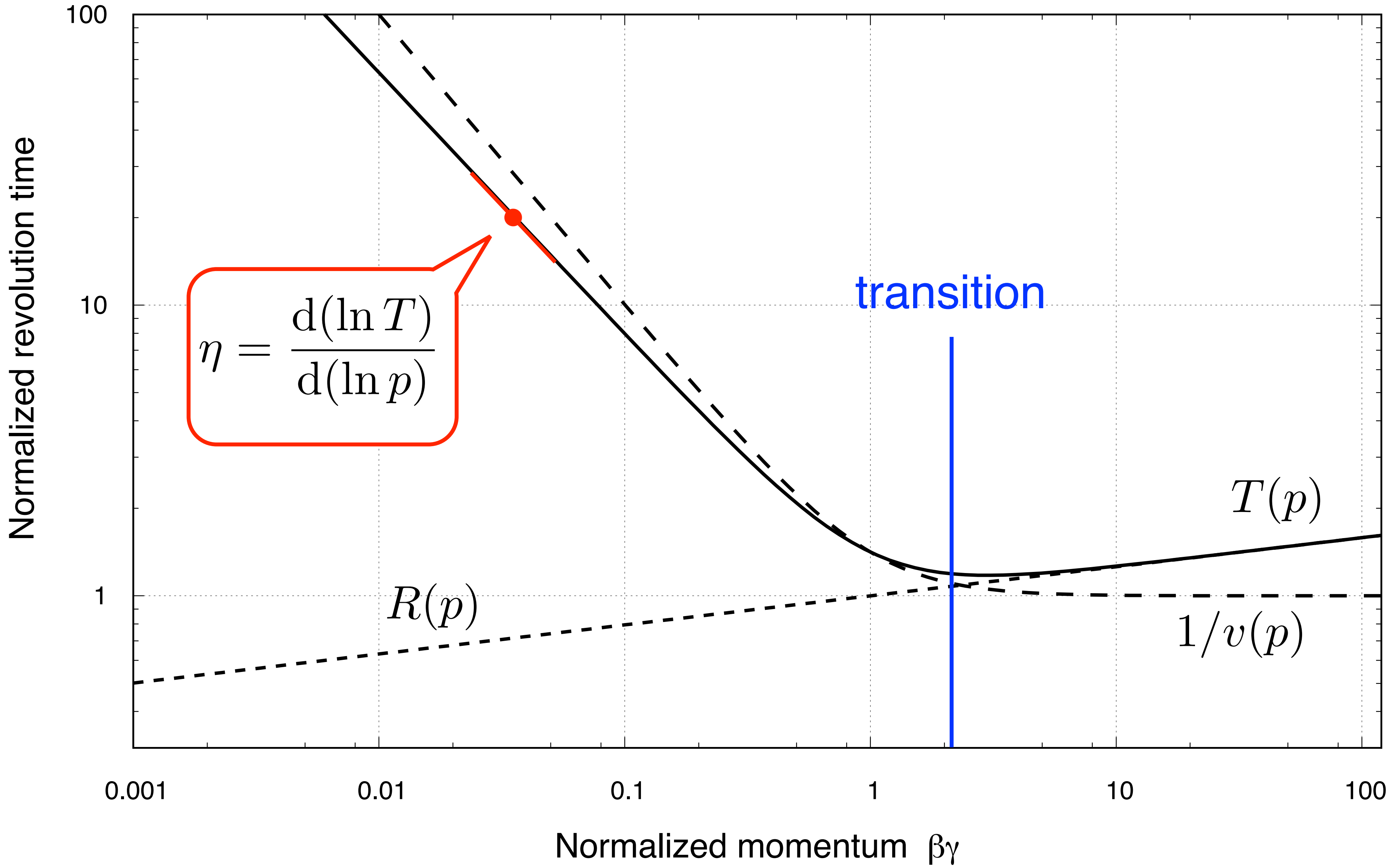
There is a minimum of T(p)
= transition



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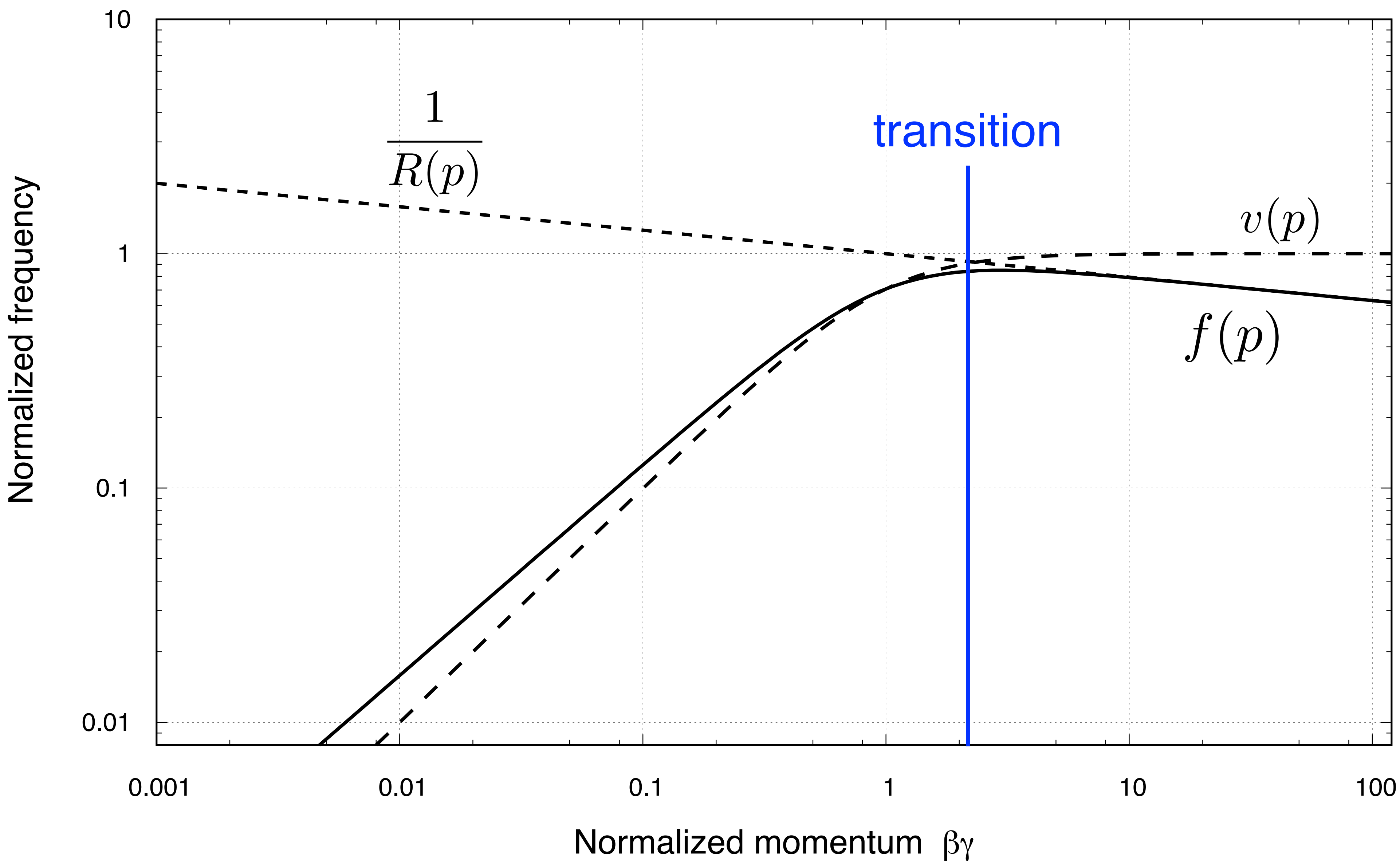

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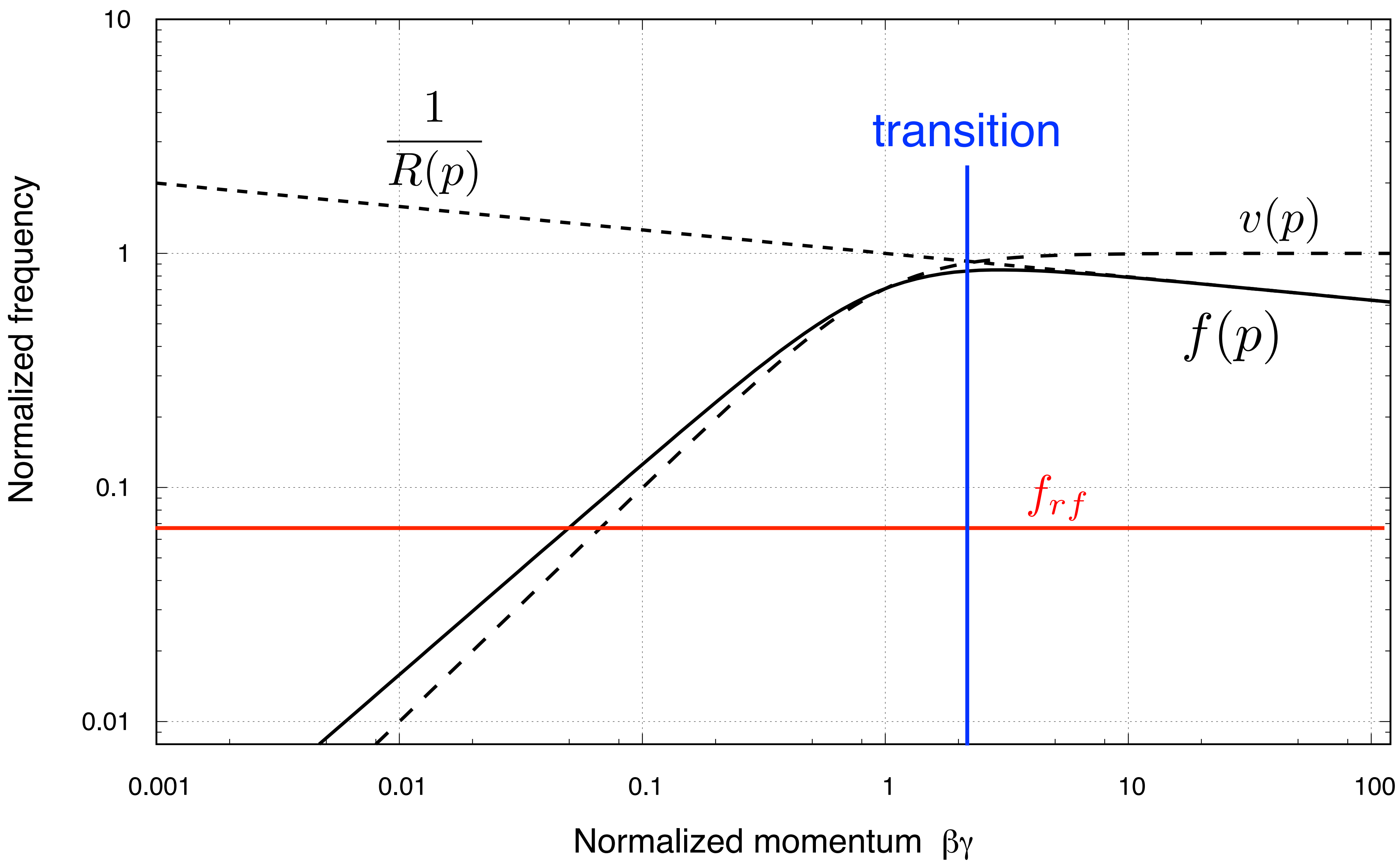

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Global function f(p) is defined in a FFAG

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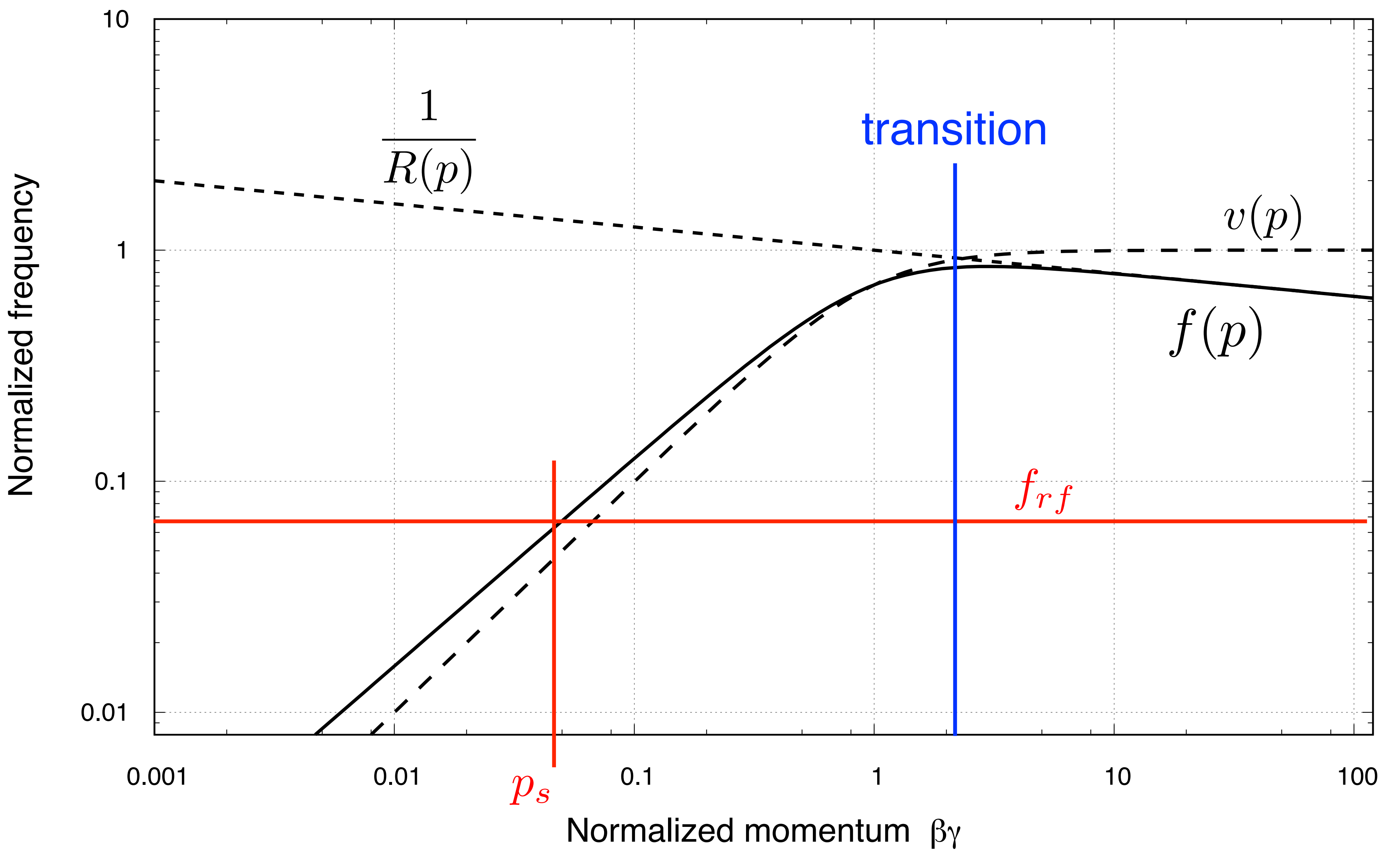
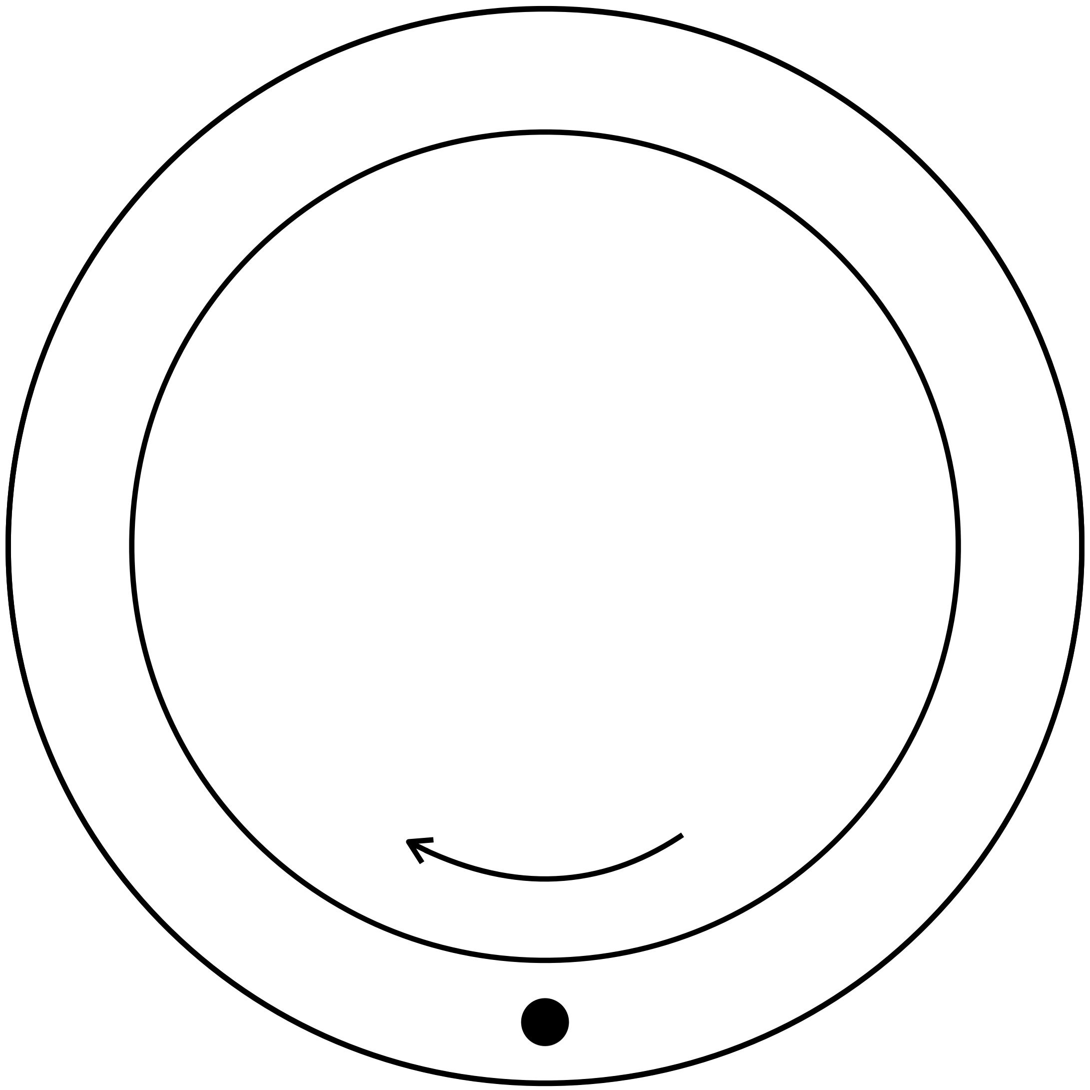


Once rf frequency is determined, 'synchronous momentum' is determined.

Longitudinal phase space

(in case $\eta < 0$)



● Particle of synchronous momentum

Longitudinal phase space

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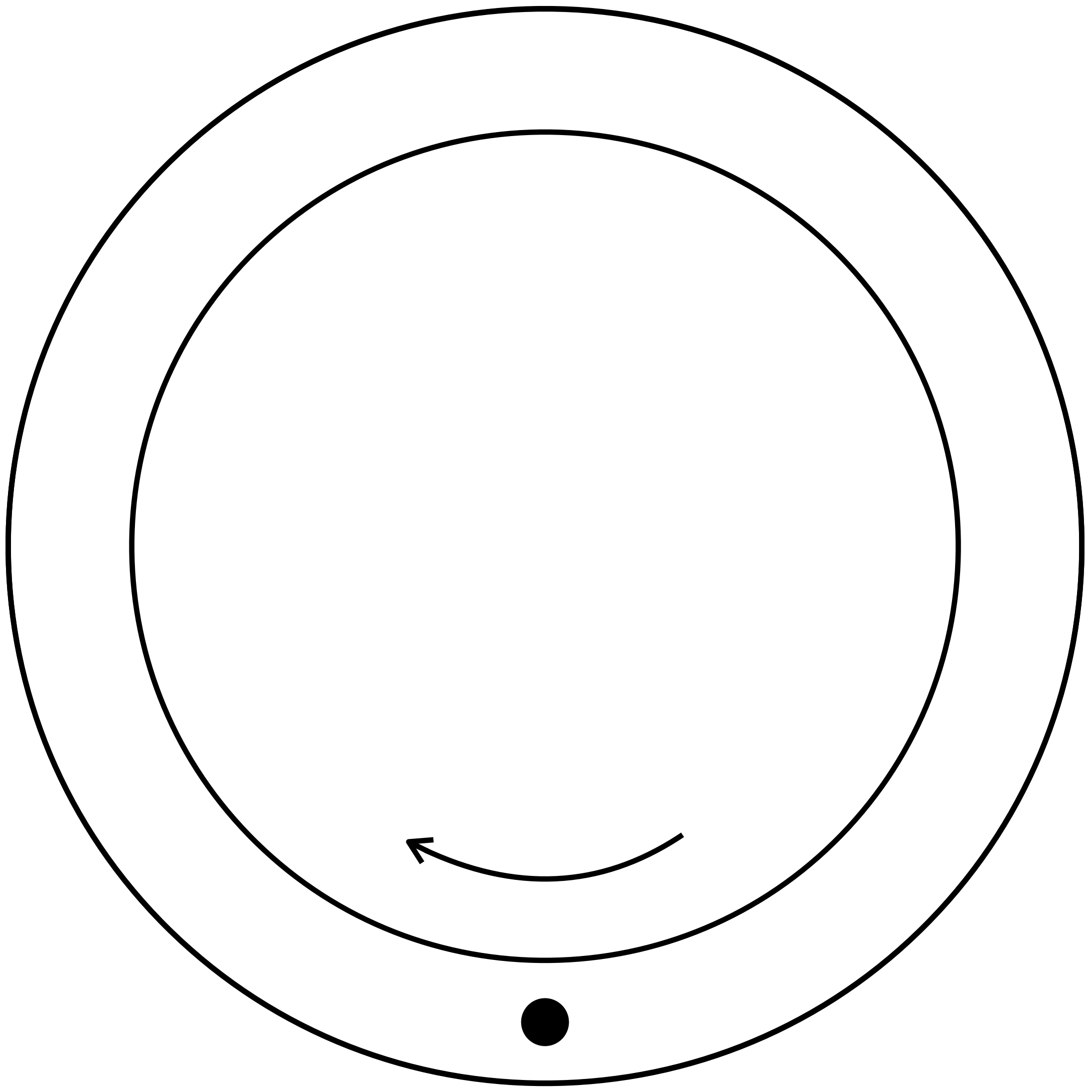

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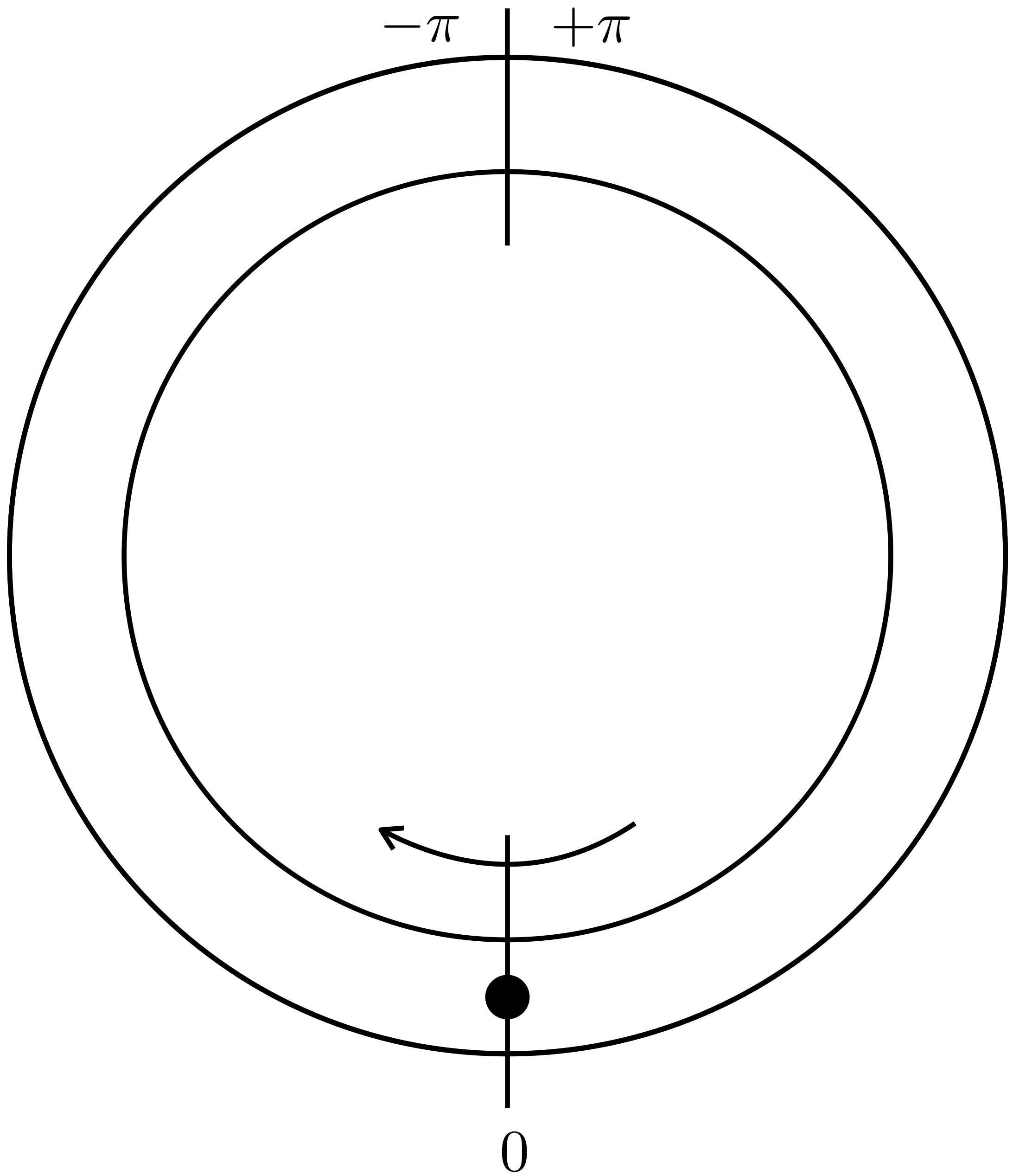

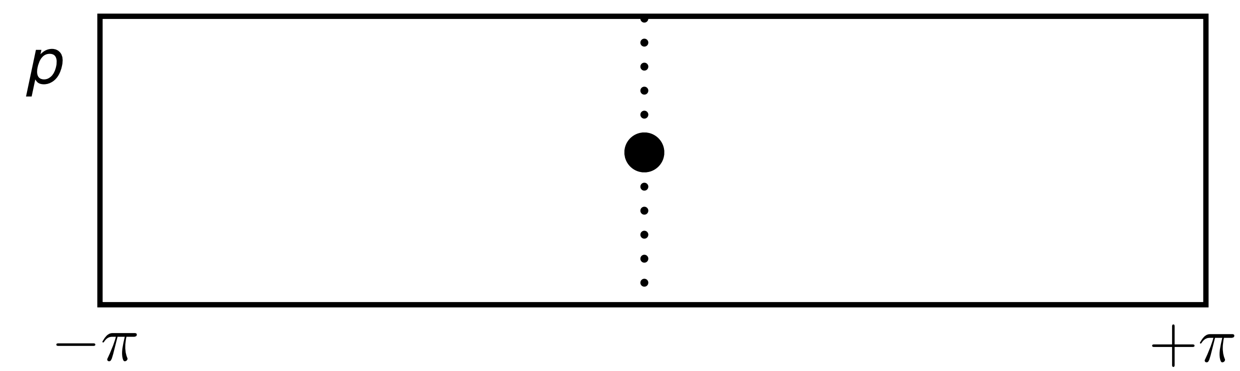

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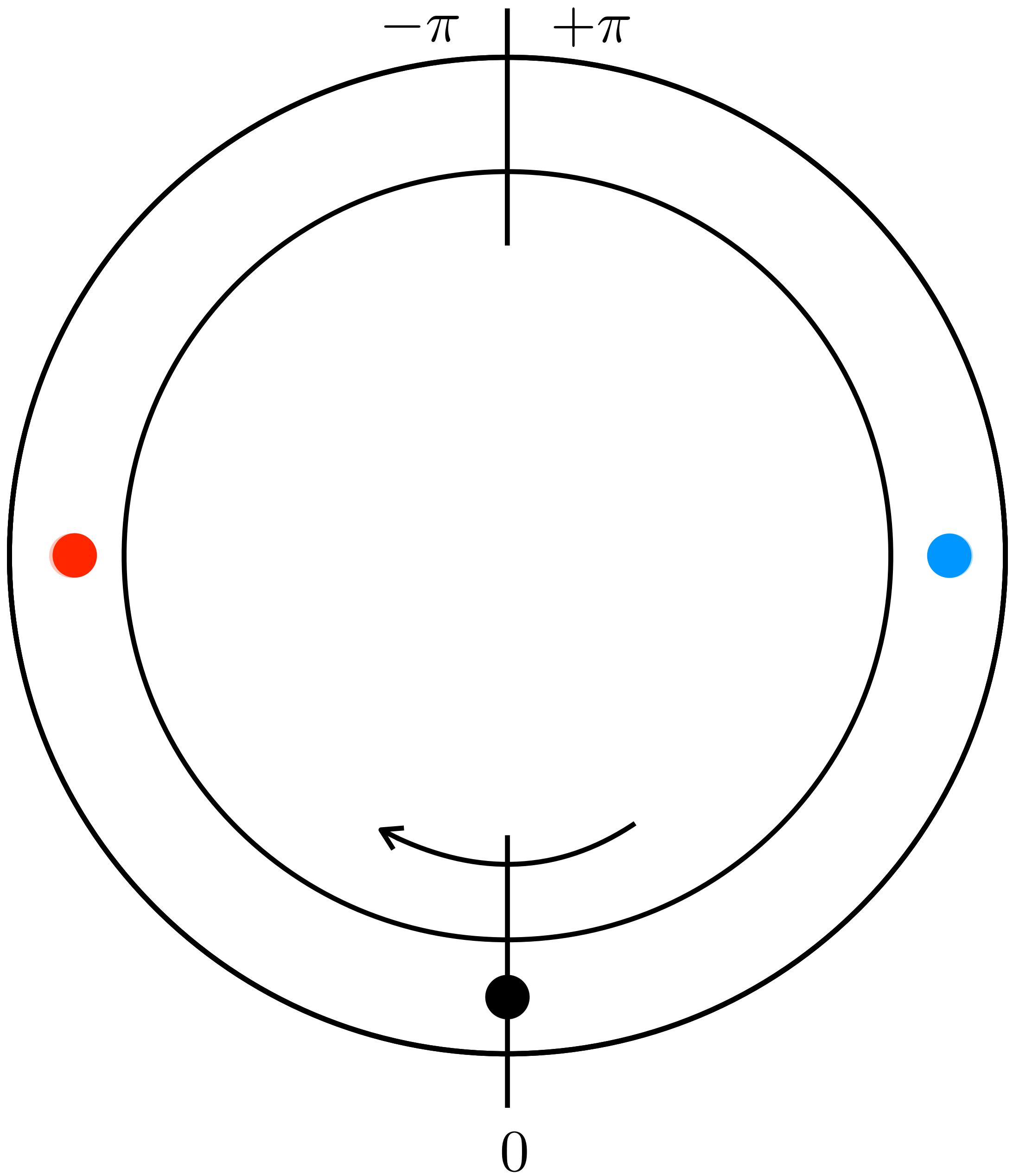

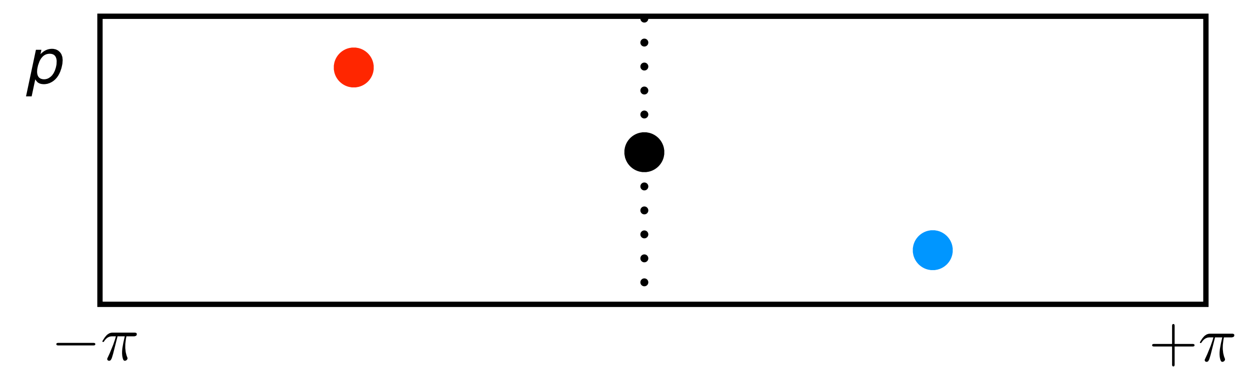

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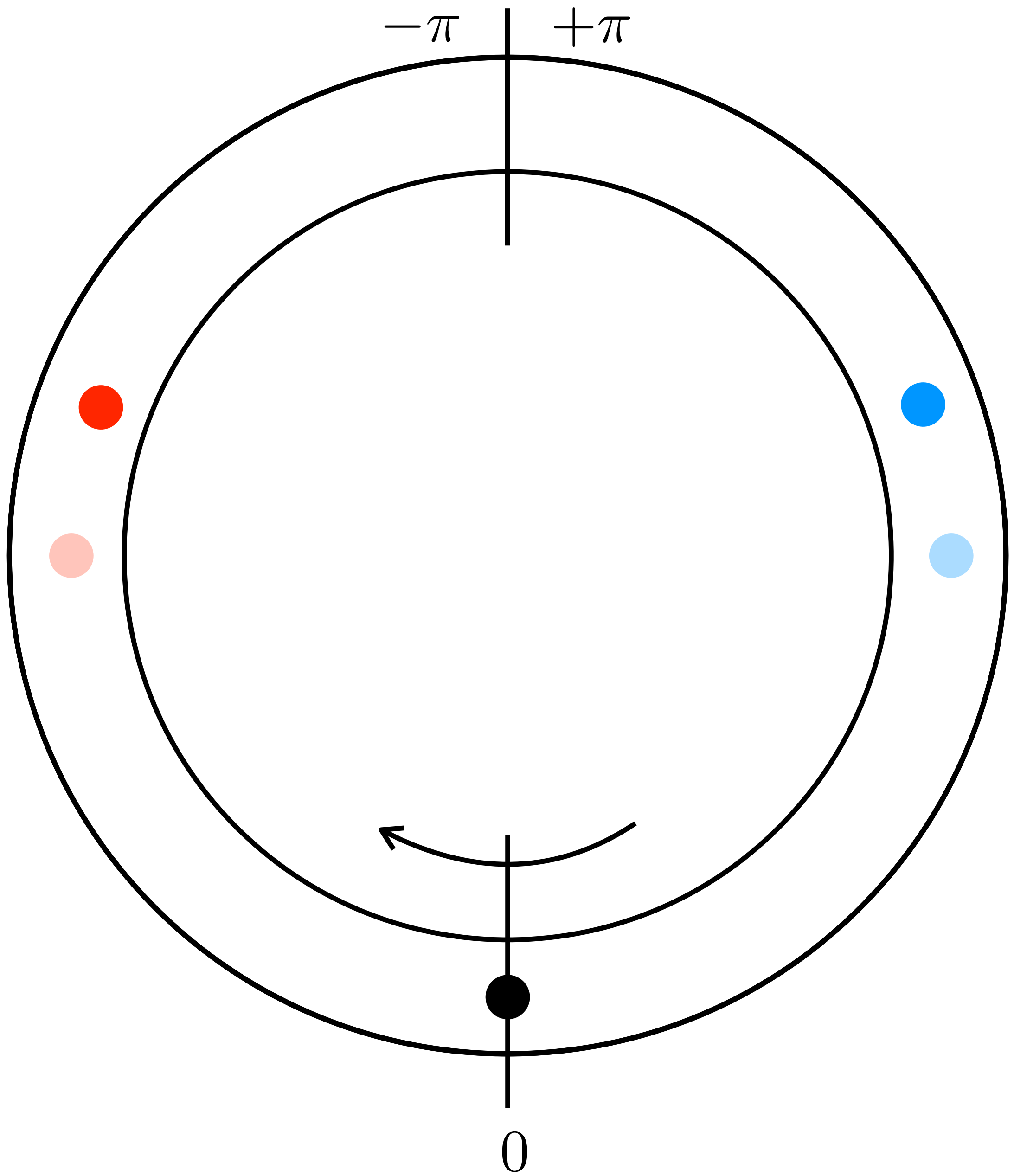

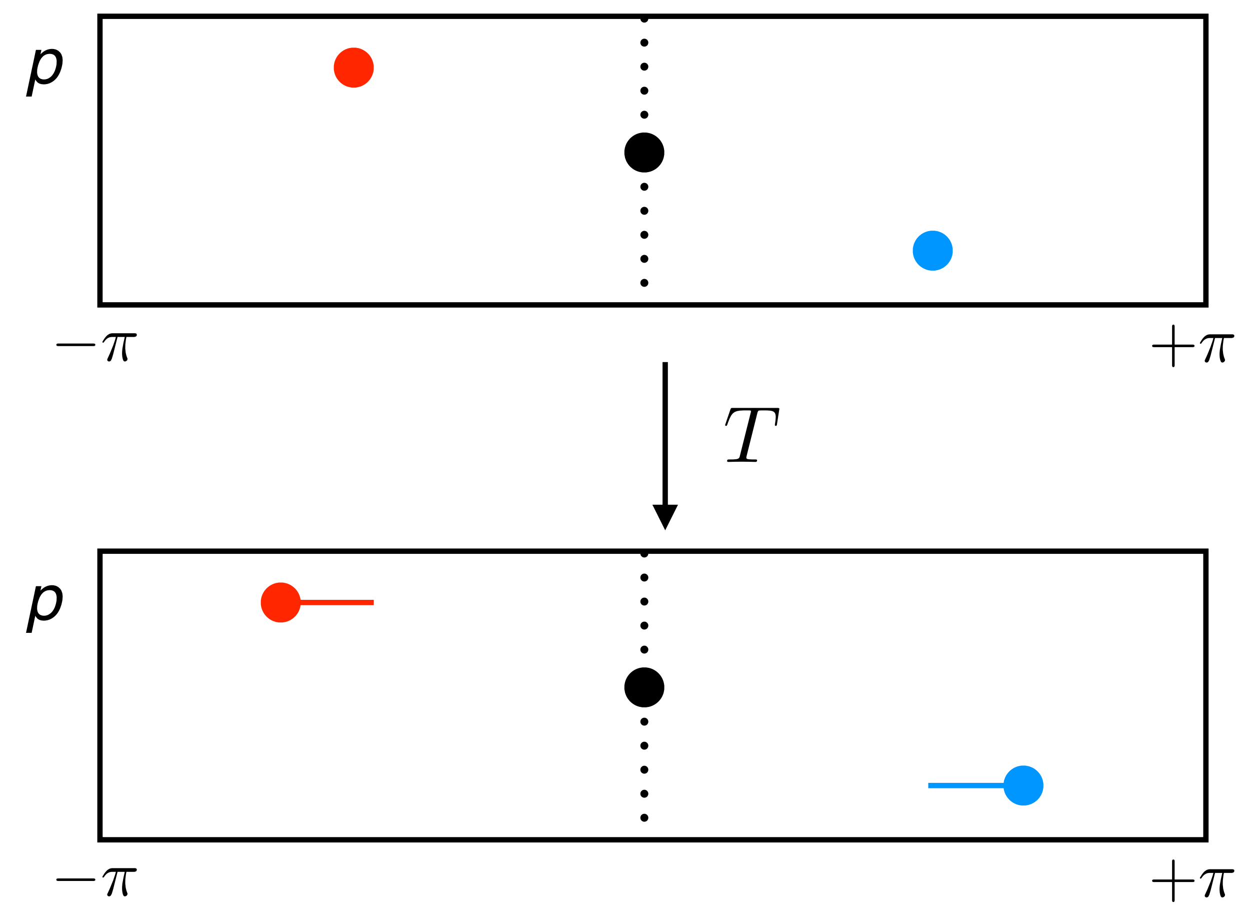

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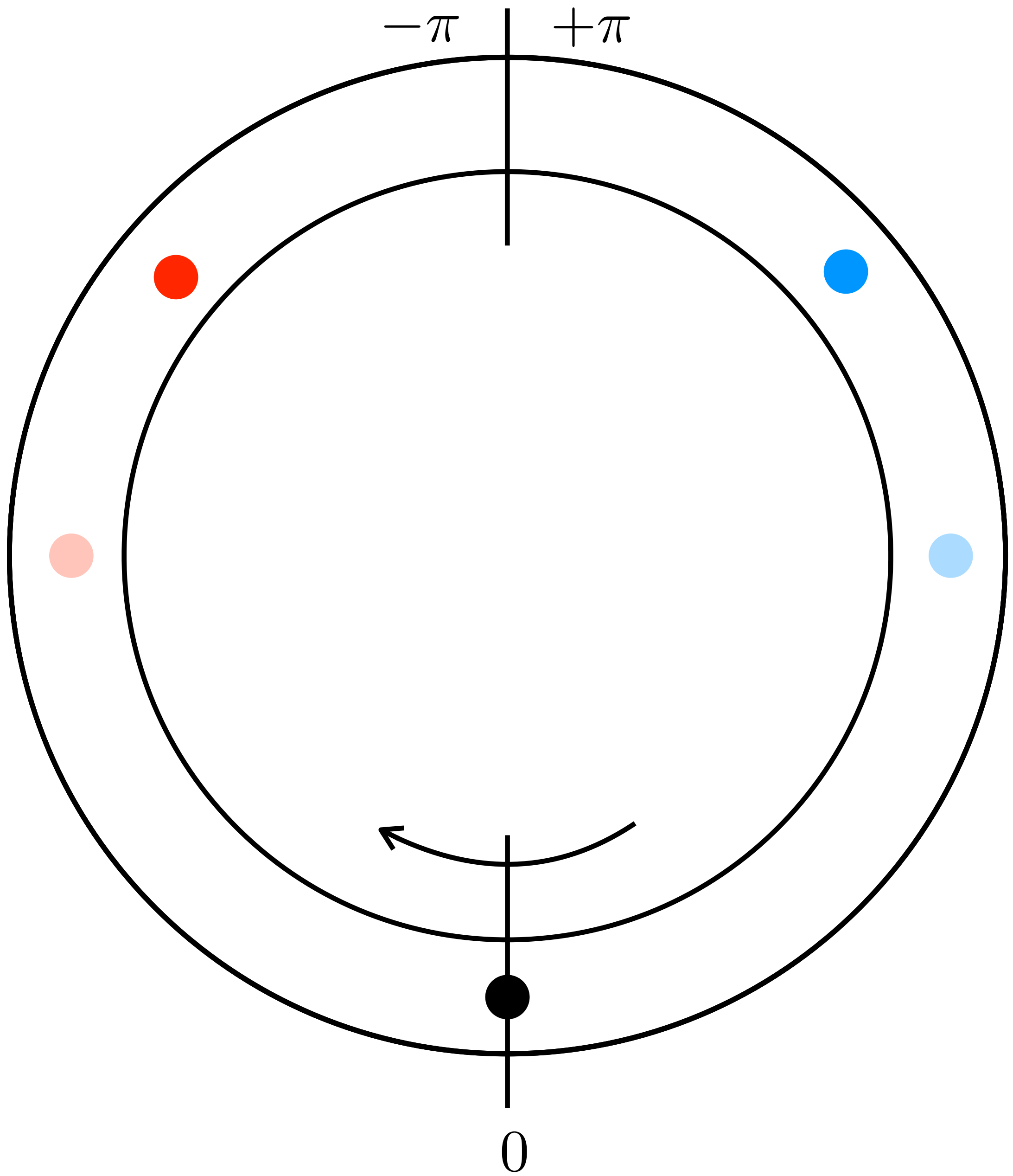

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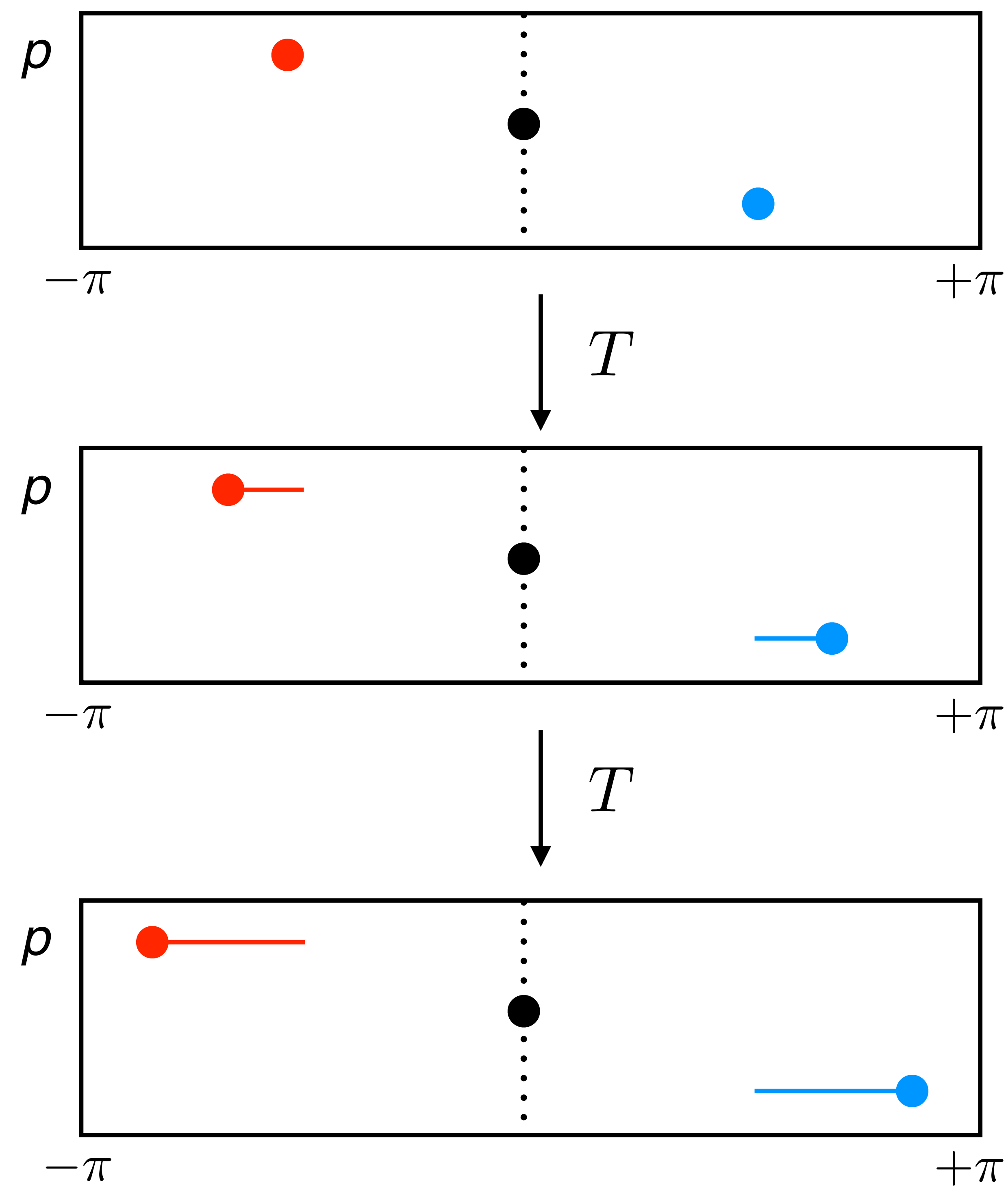
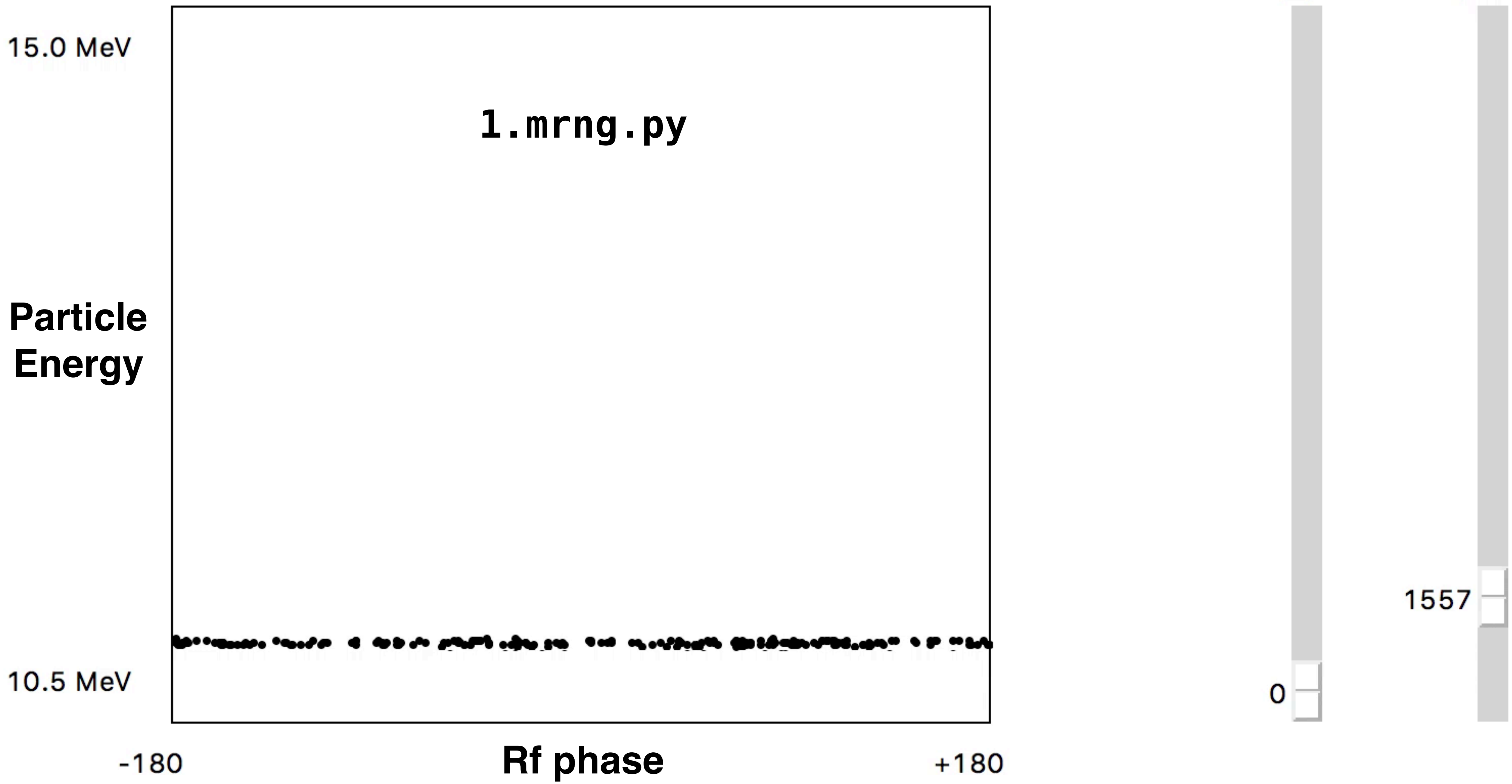


Demonstration 1: Slippage

QUIT

$$m = 938\ MeV \qquad \alpha_p = \frac{1}{7.6+1}$$

$$f = 1591.84\ MHz \quad at\ E_k = 11.57\ MeV$$



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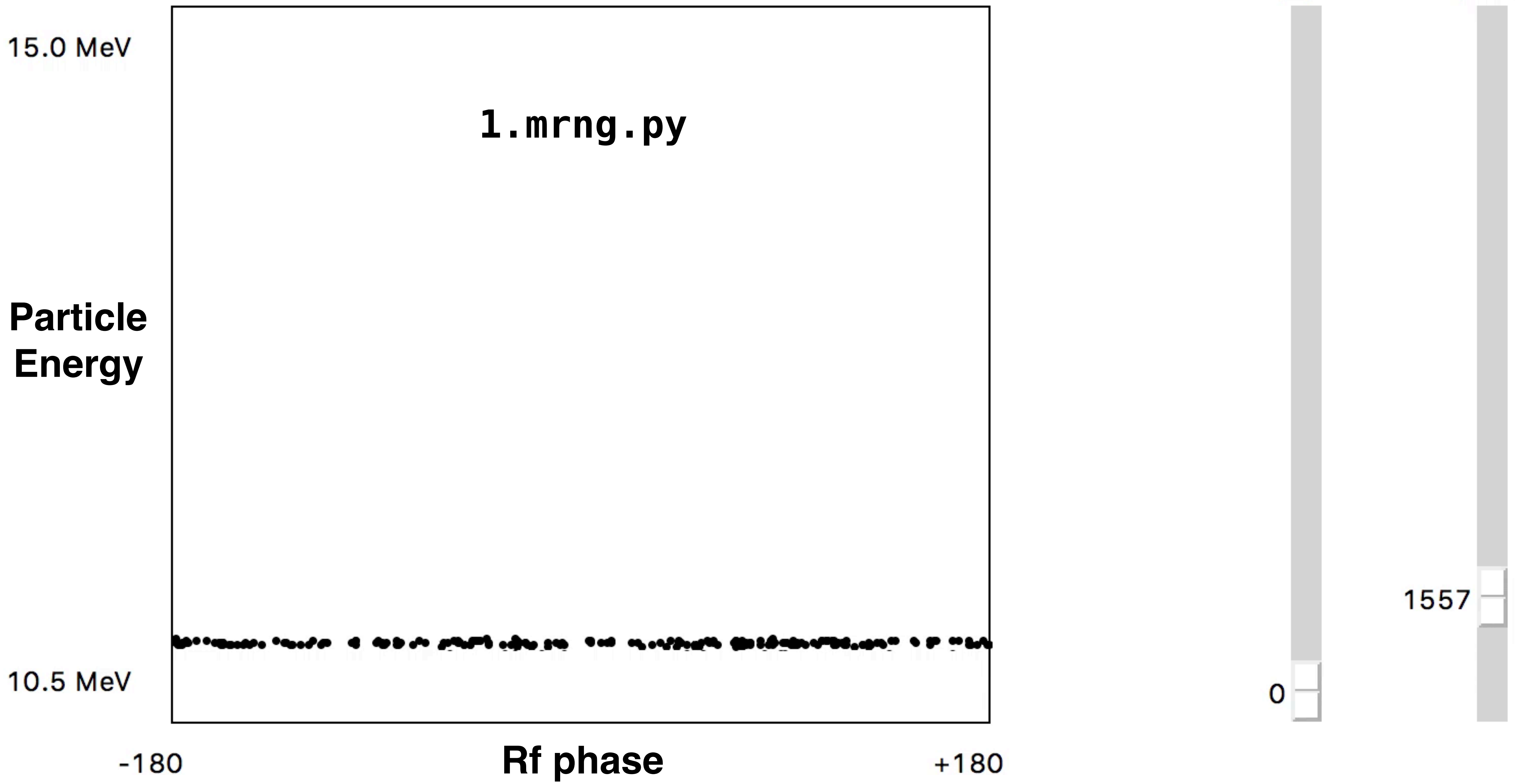

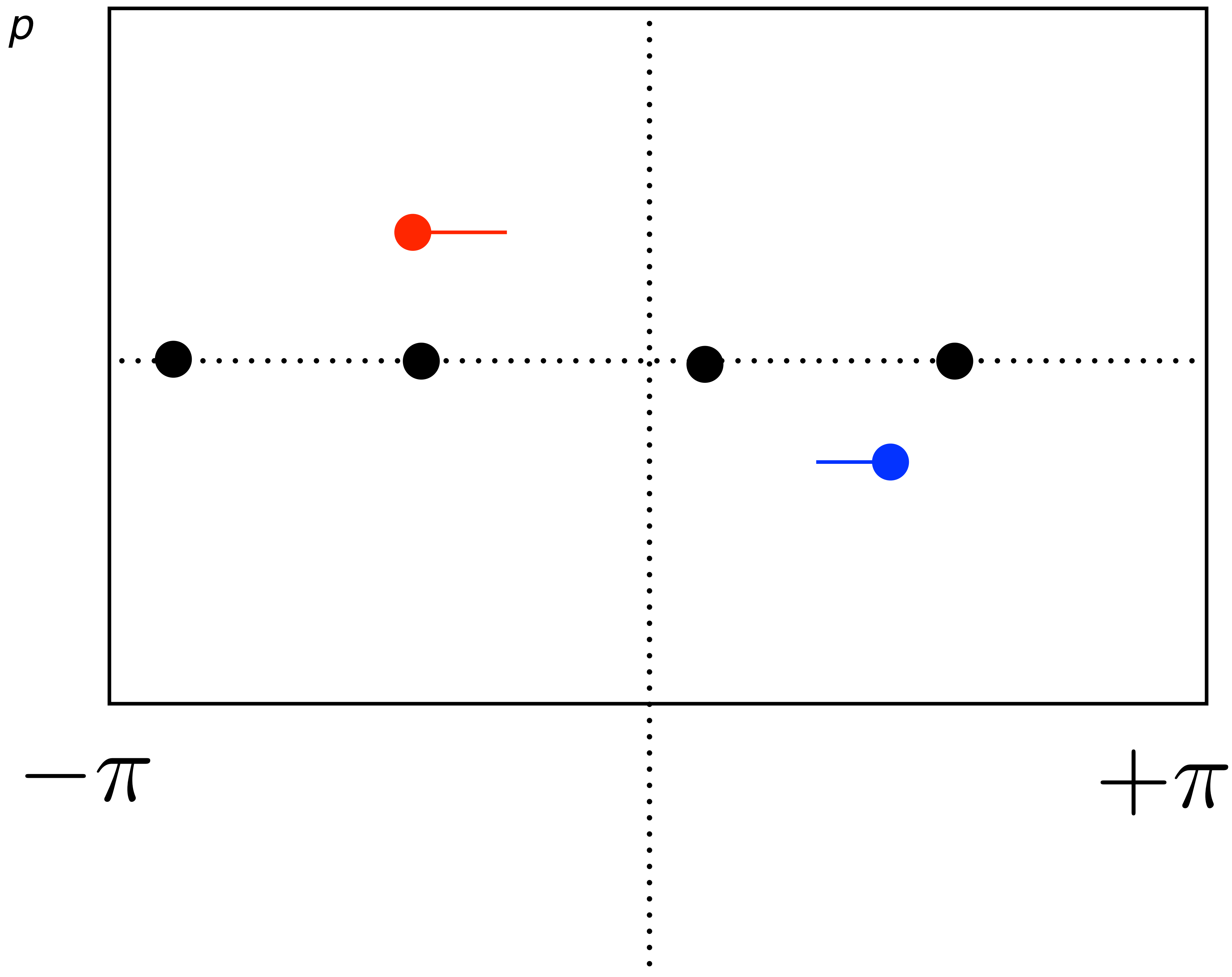

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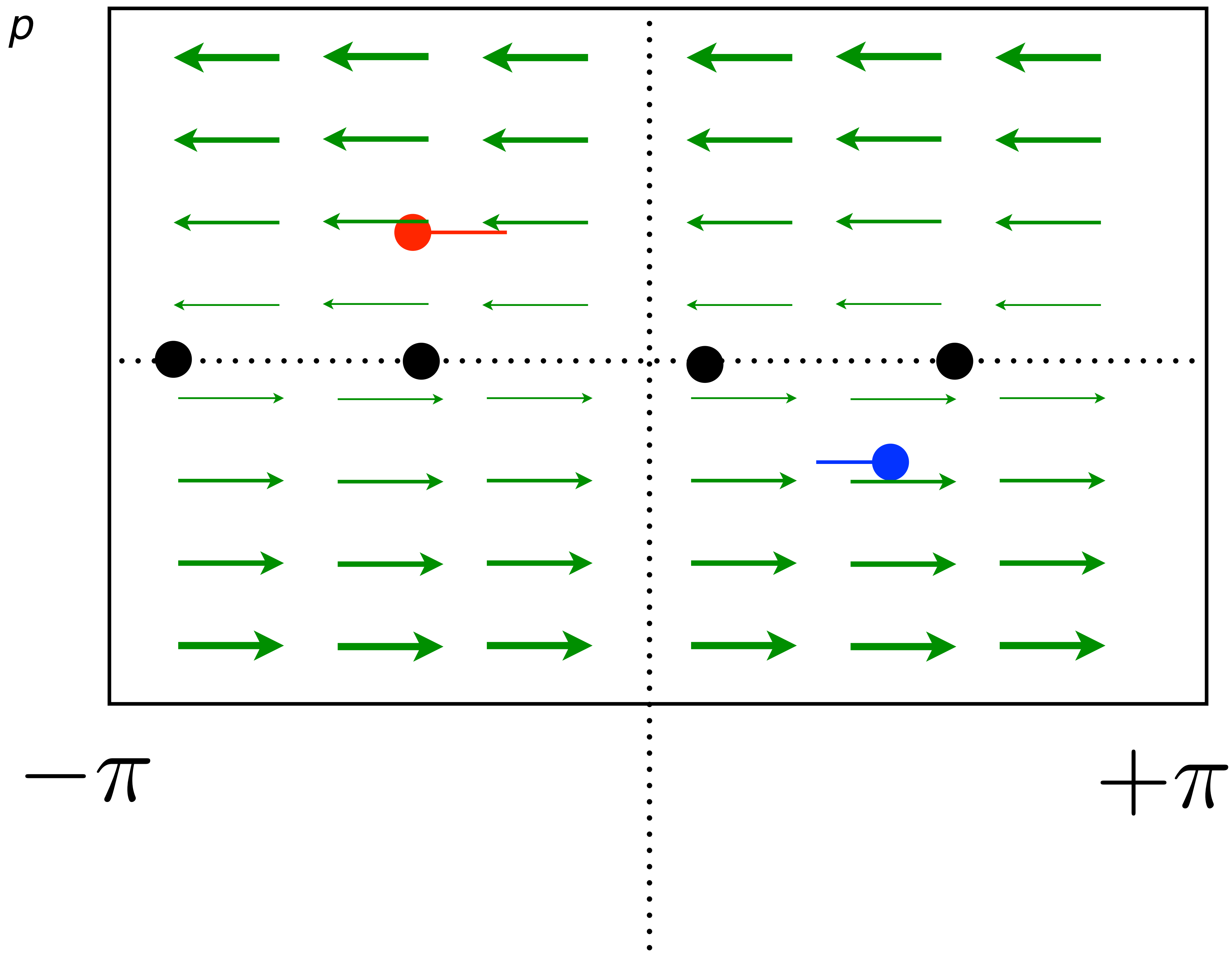

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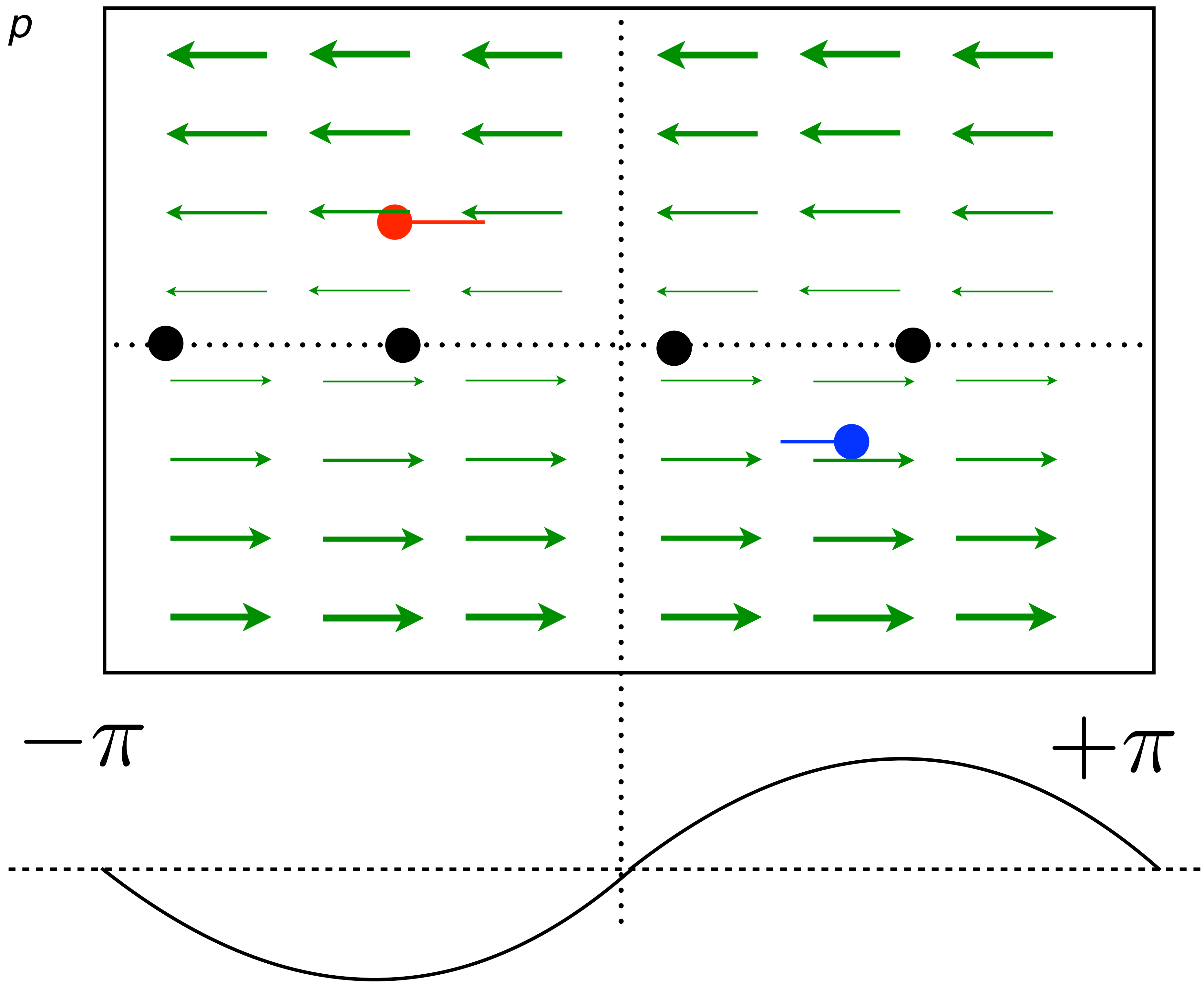
Rf voltage is applied



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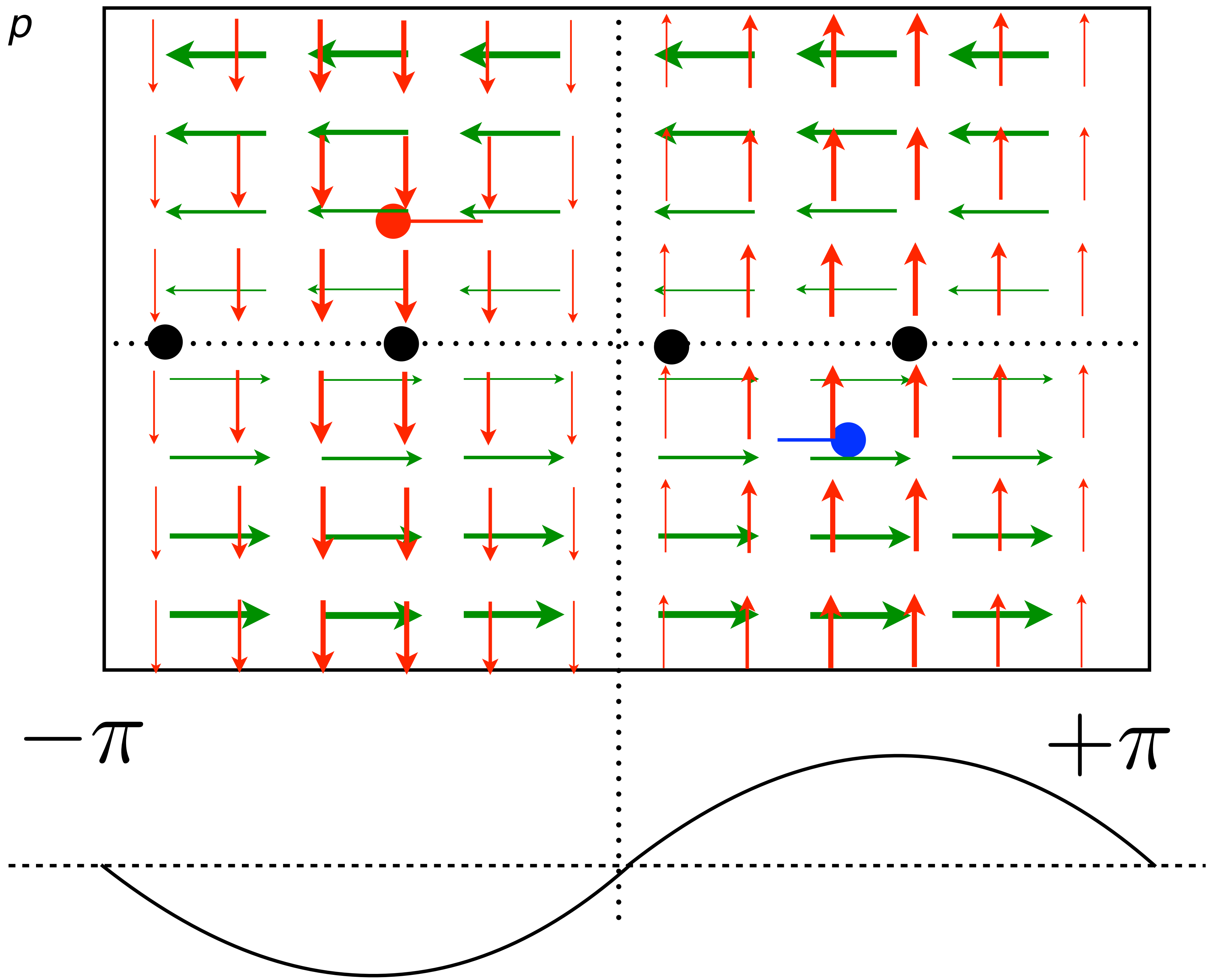

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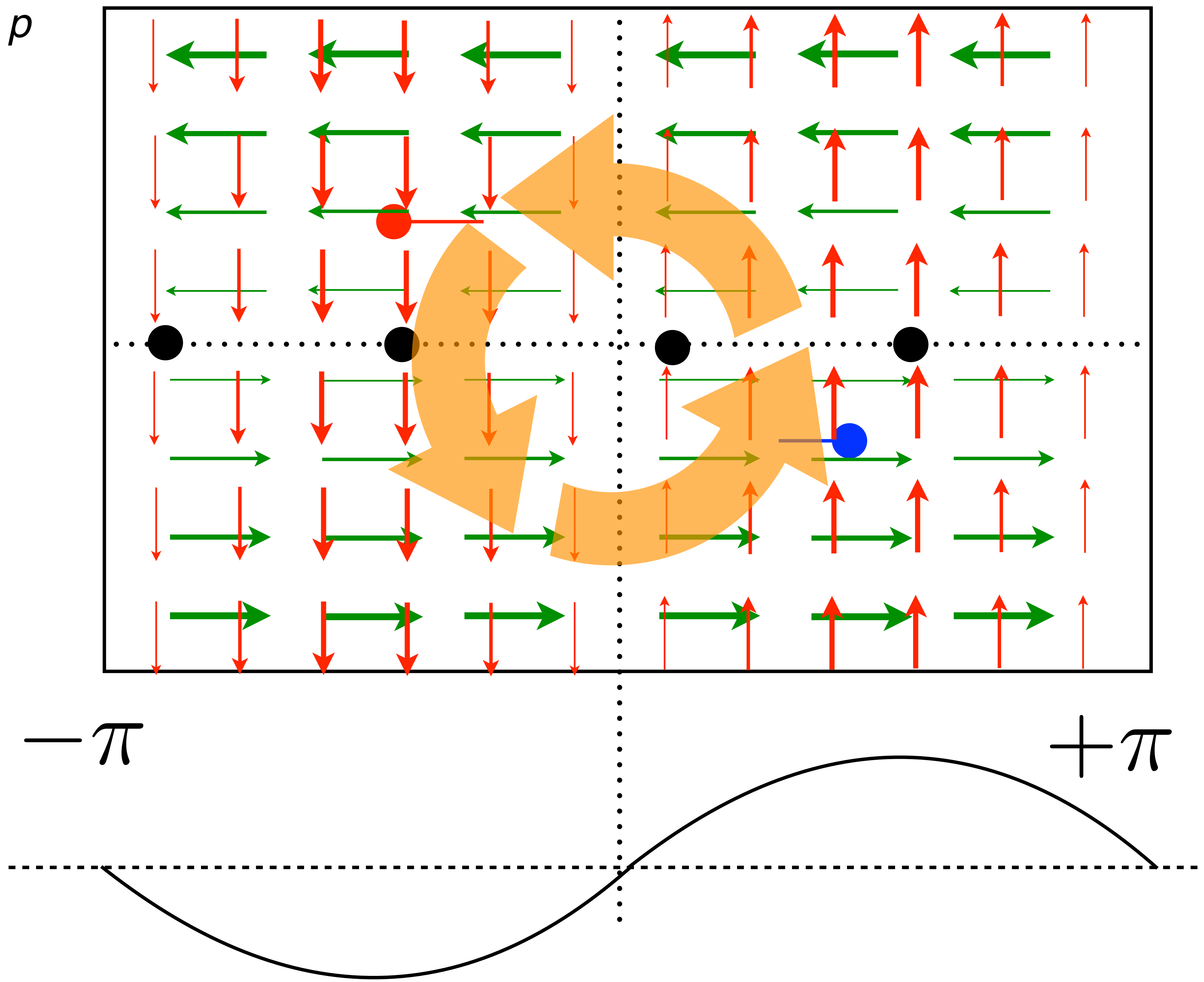

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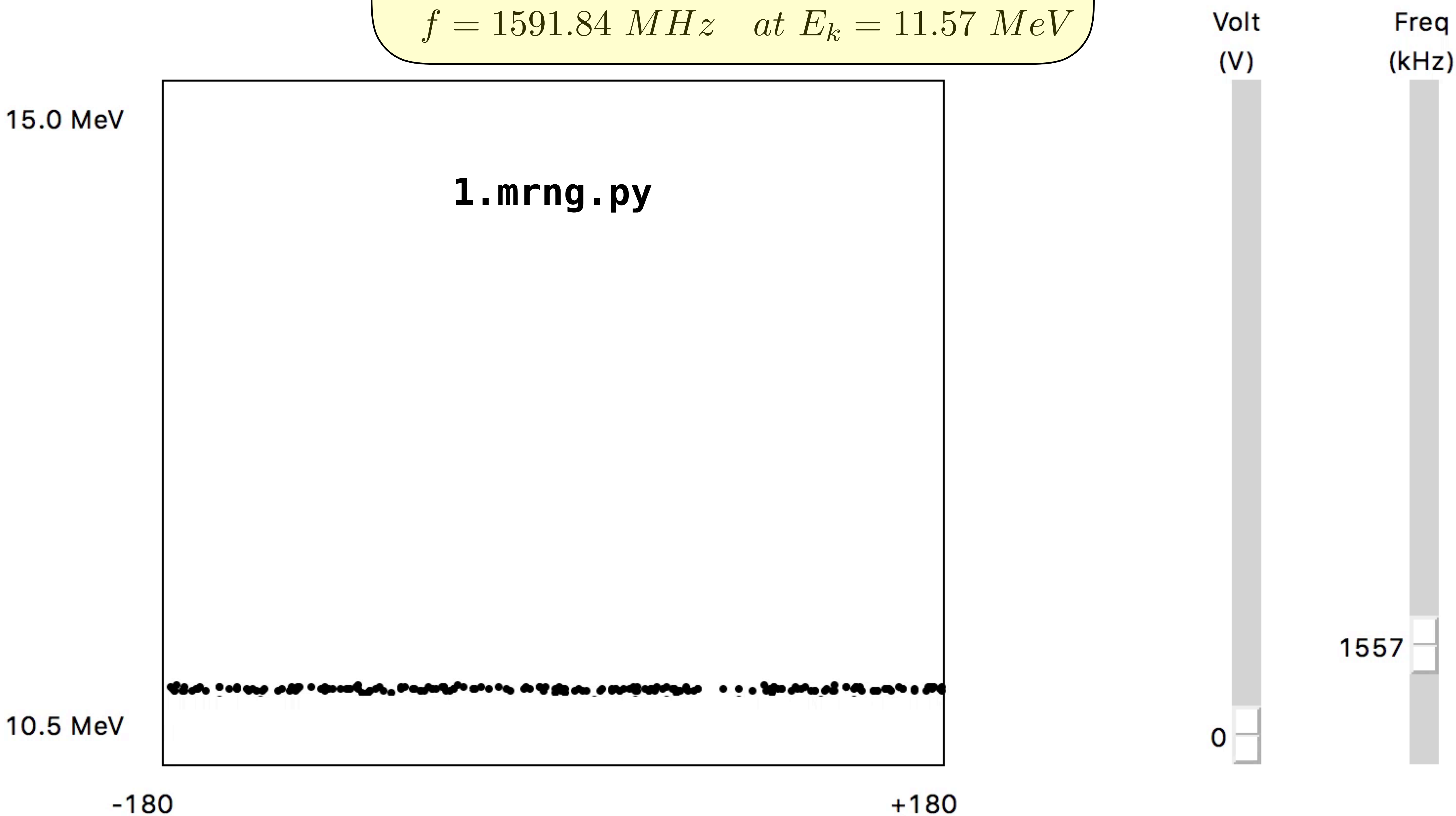

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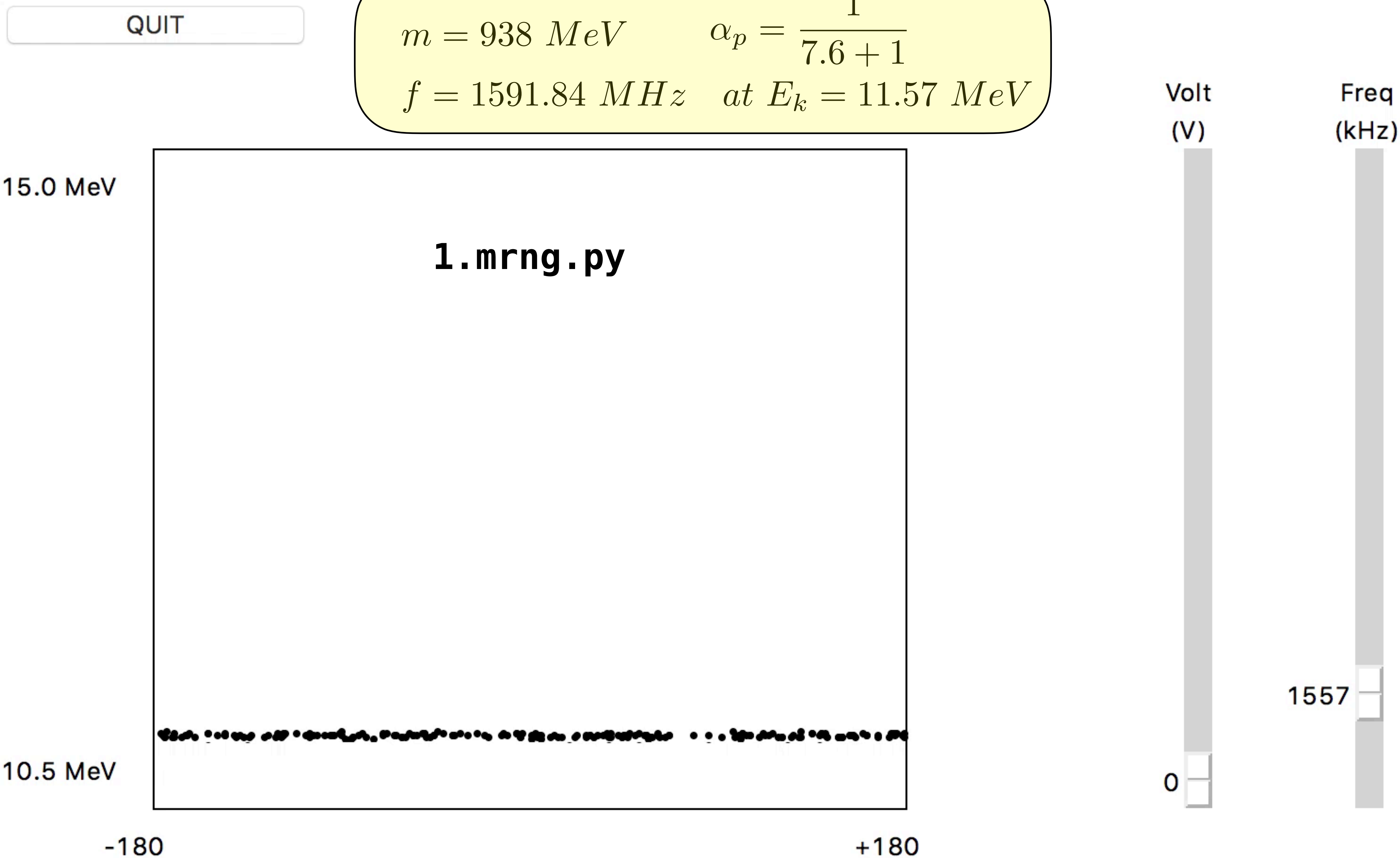
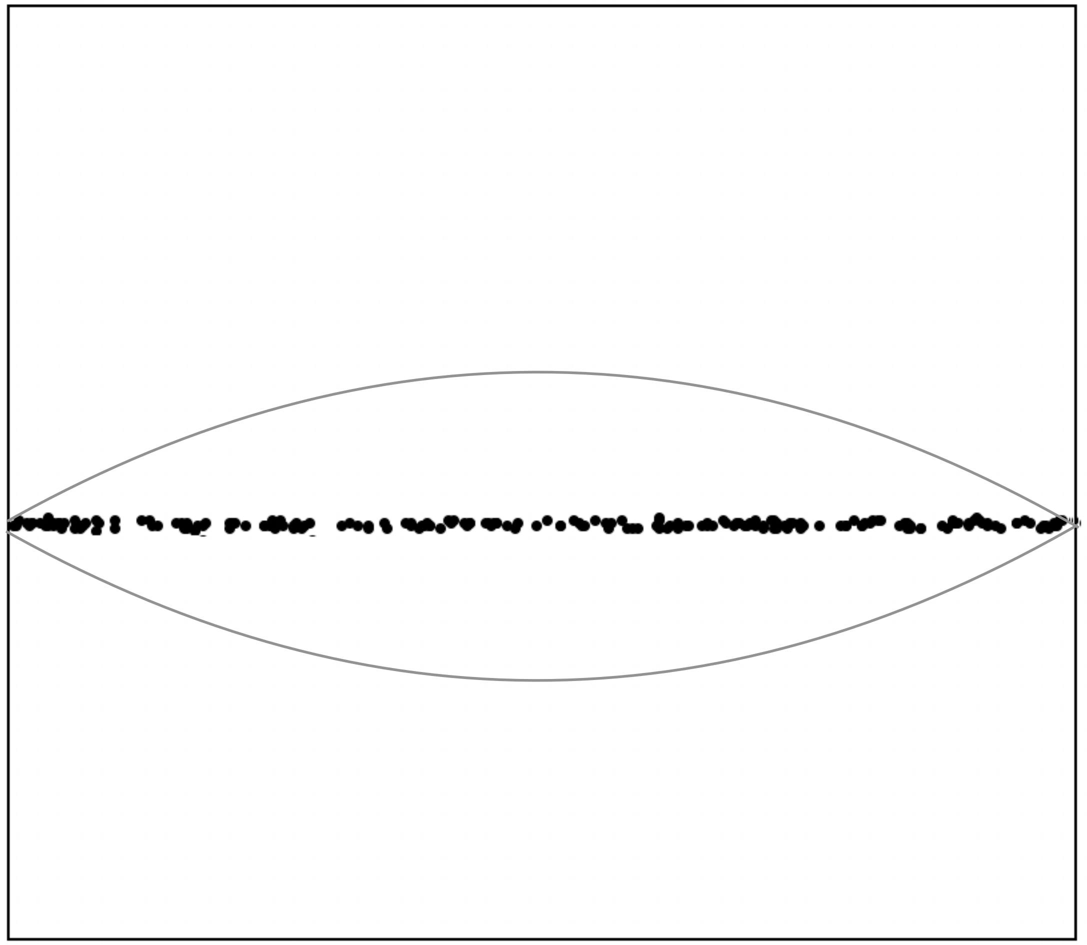


synchronous energy

Remark; Filamentation

If the injected beam is *mismatched* in phase space, then…



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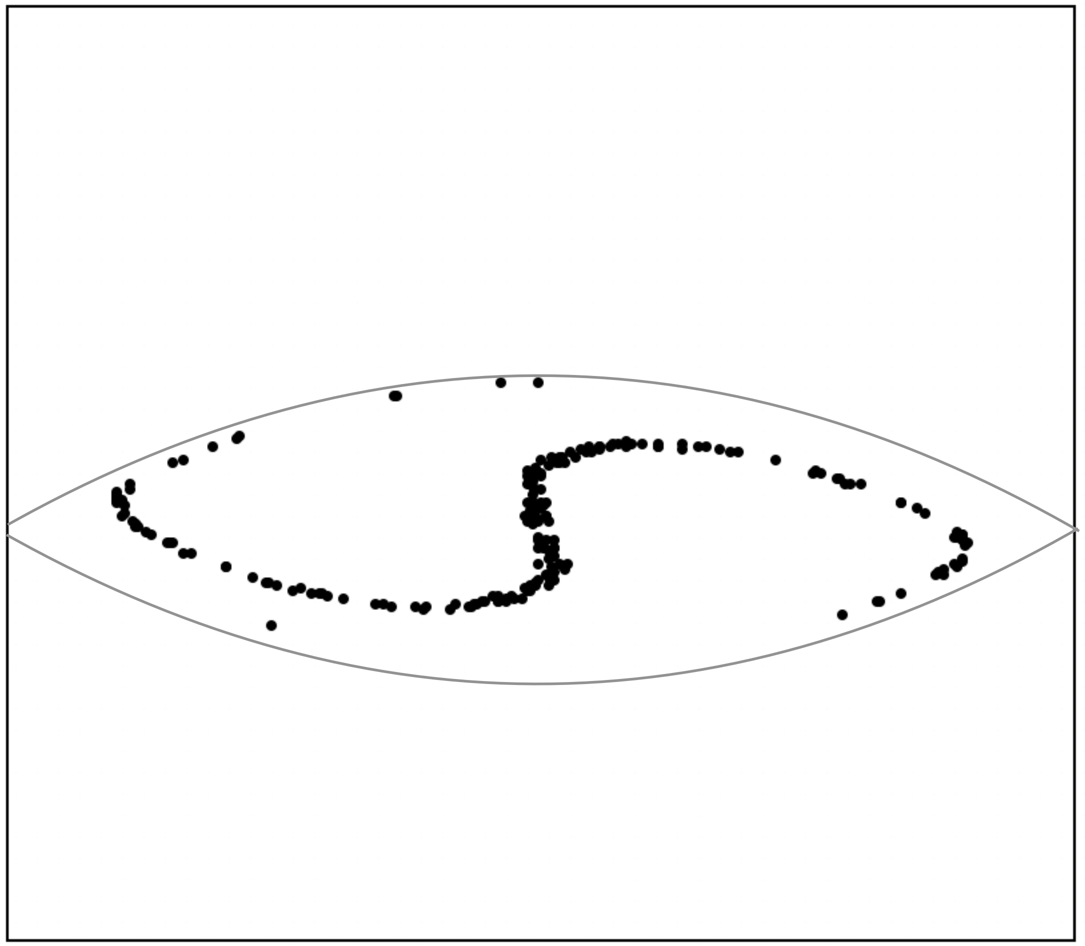

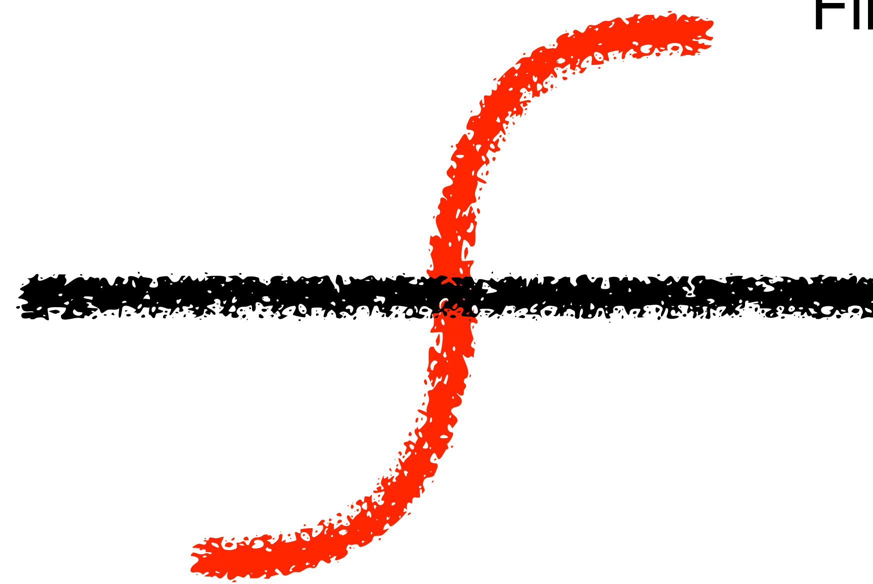

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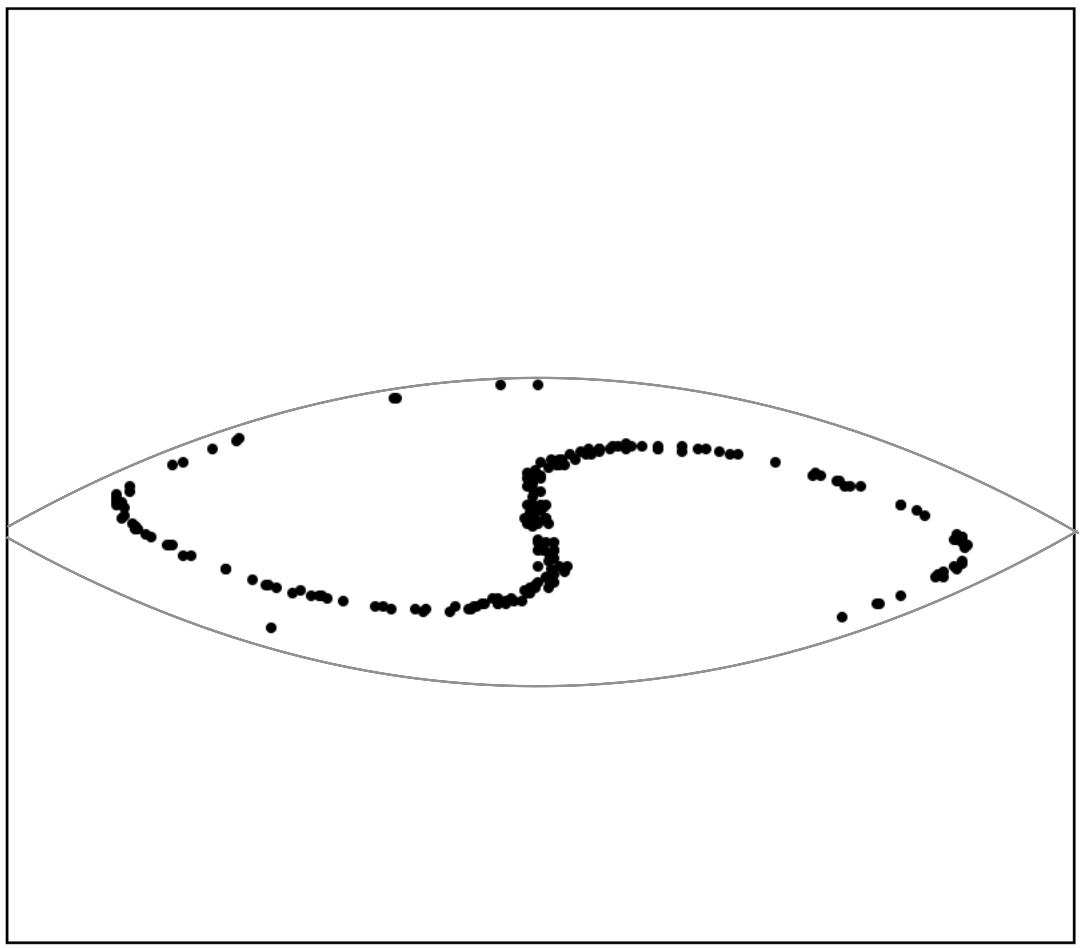

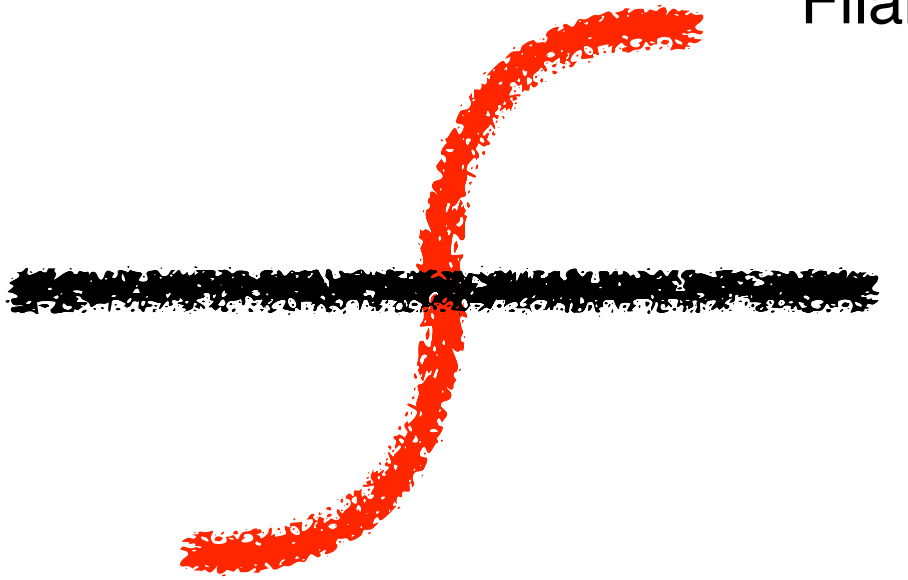

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Filamentation does not happen with linear waveform (Saw-tooth) rf.

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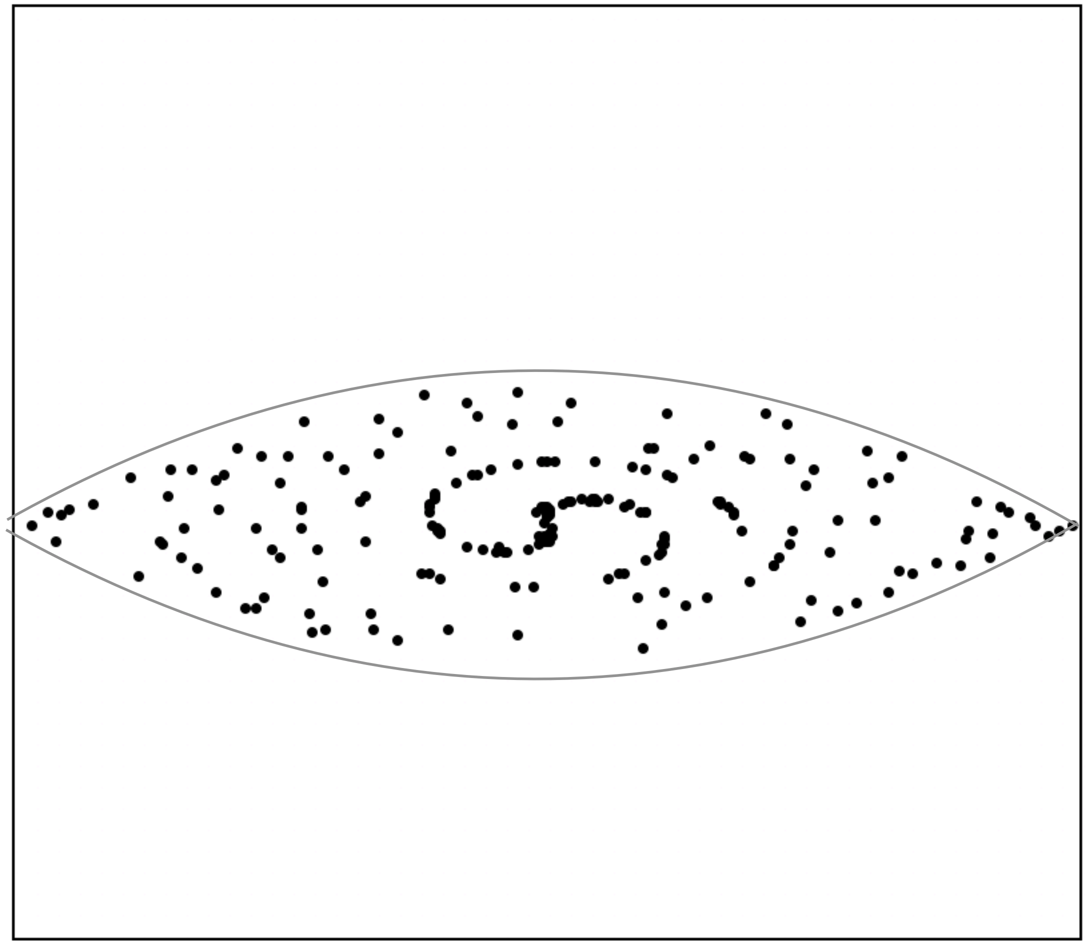

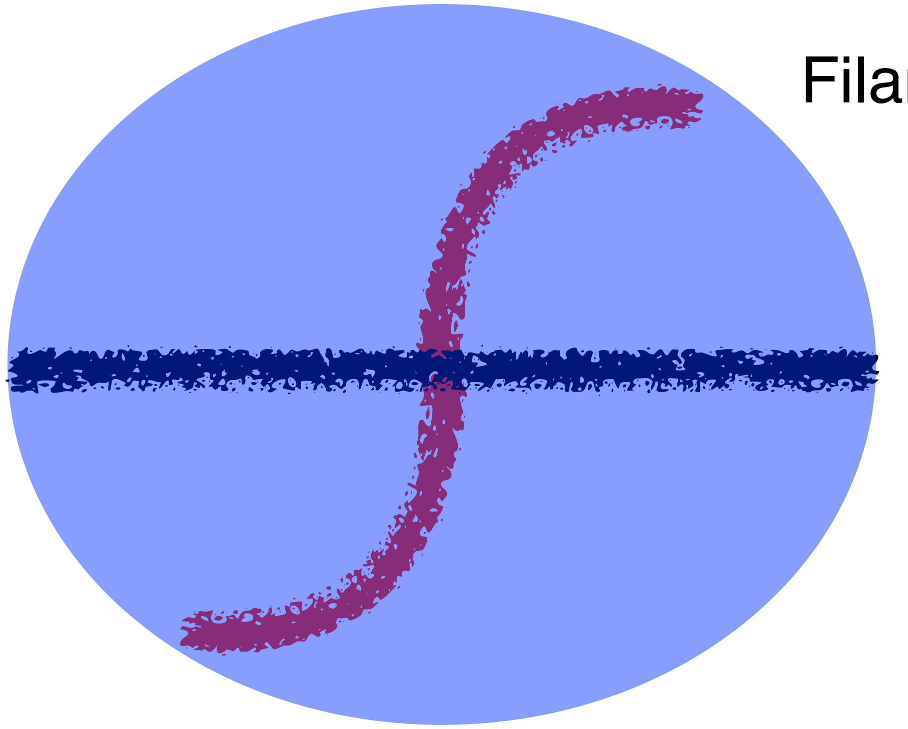

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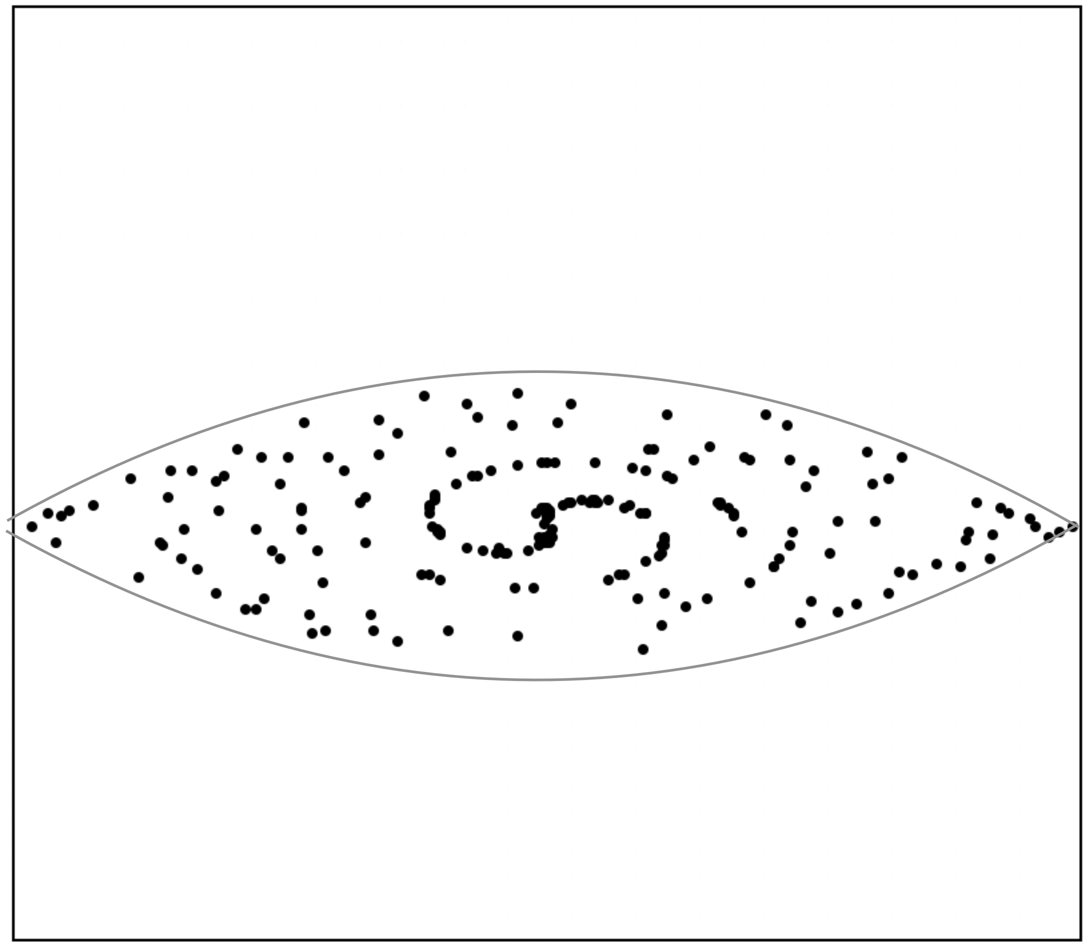

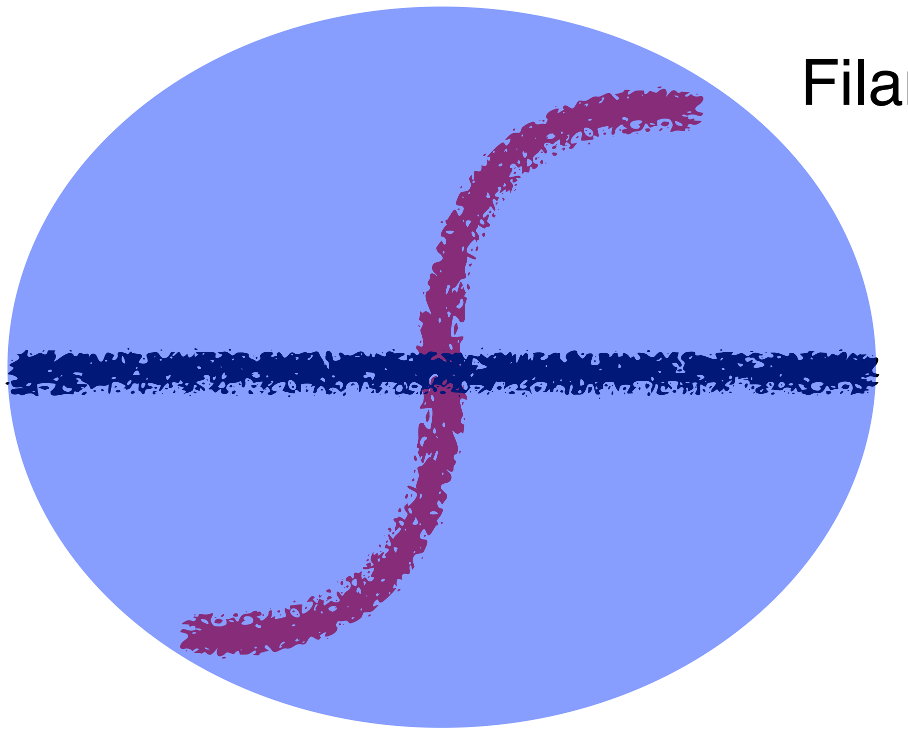

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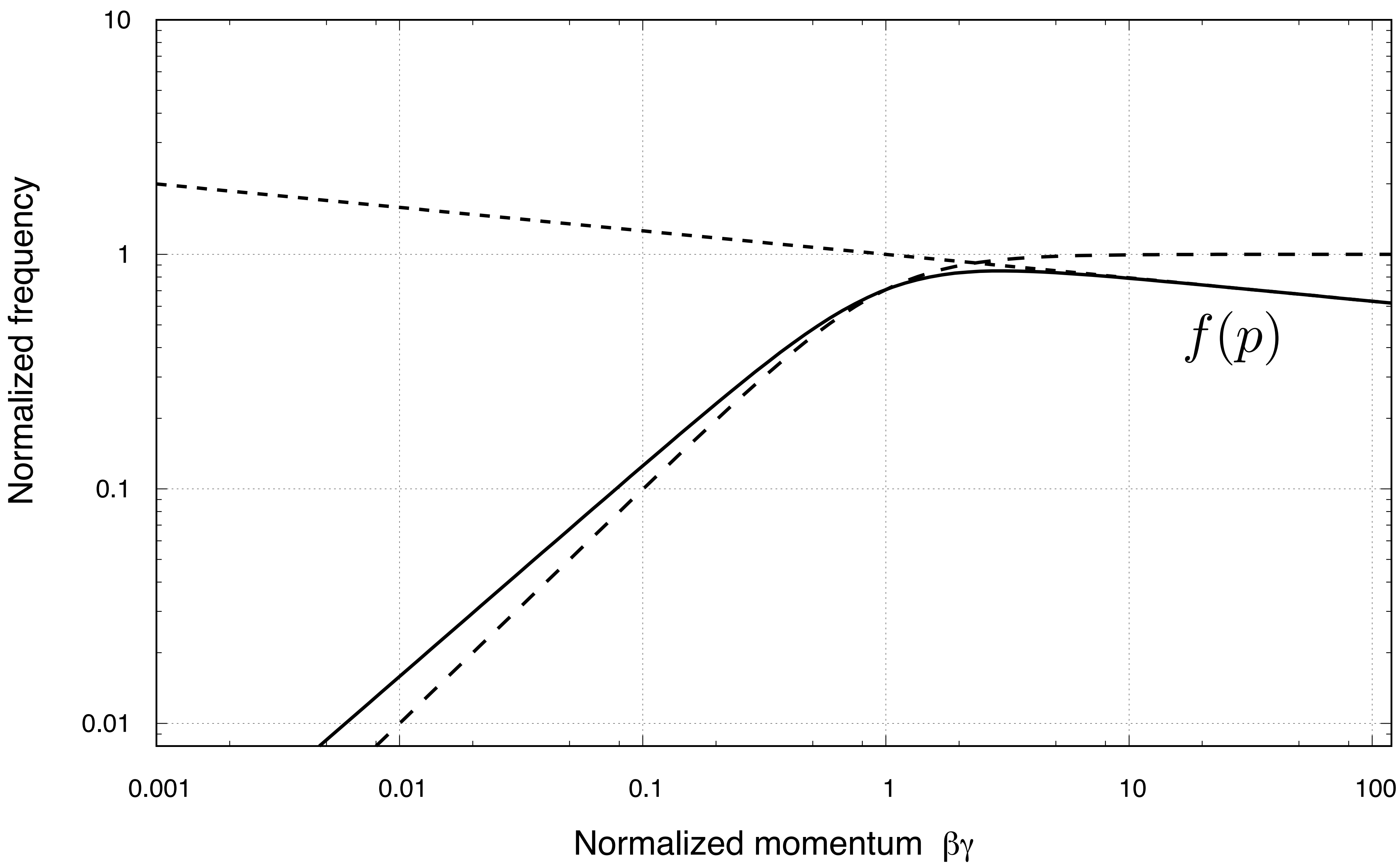
Emittance growth caused by filamentation is reduced by adiabatic capture.

Acceleration

is done by slowly changing the rf frequency



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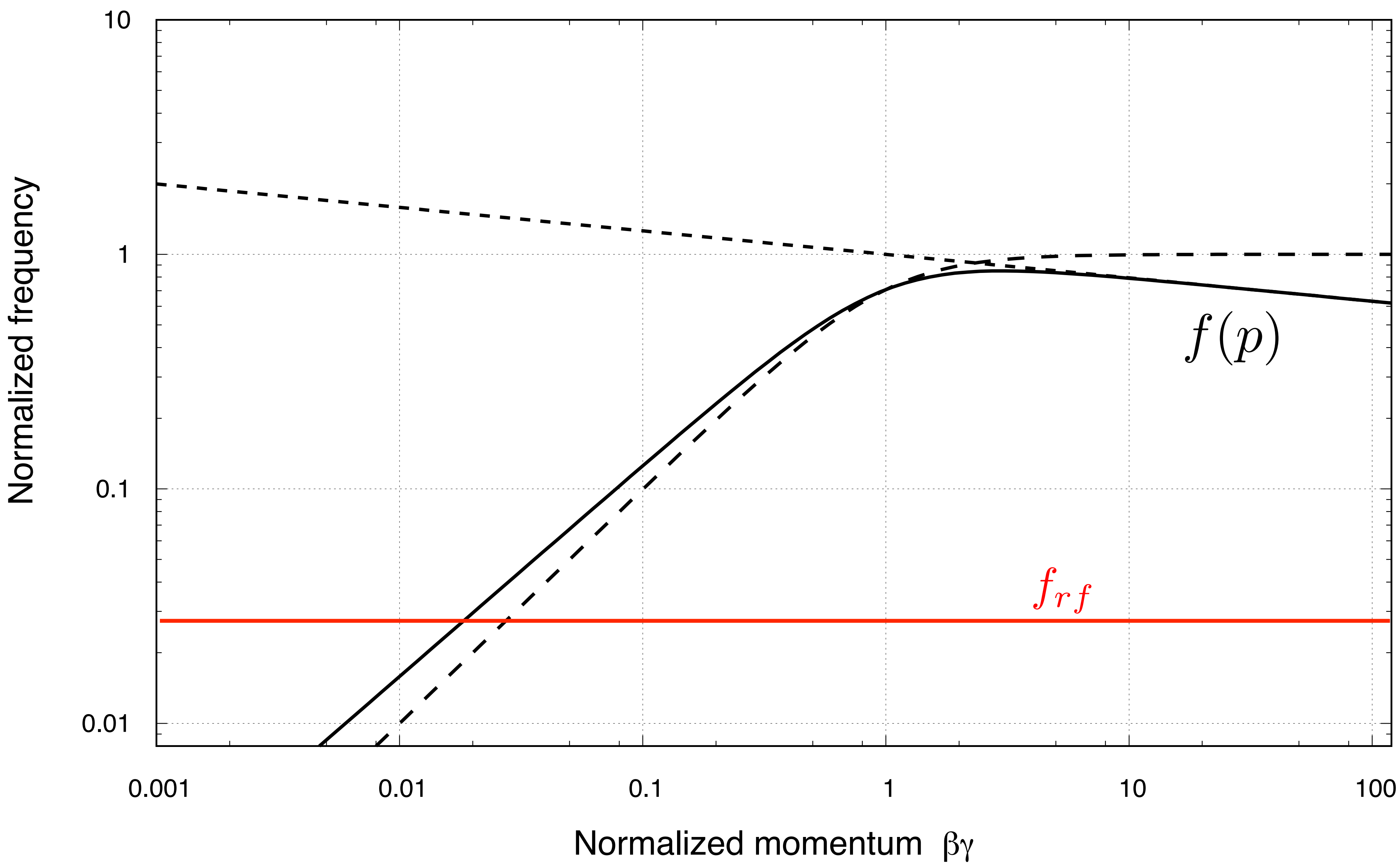

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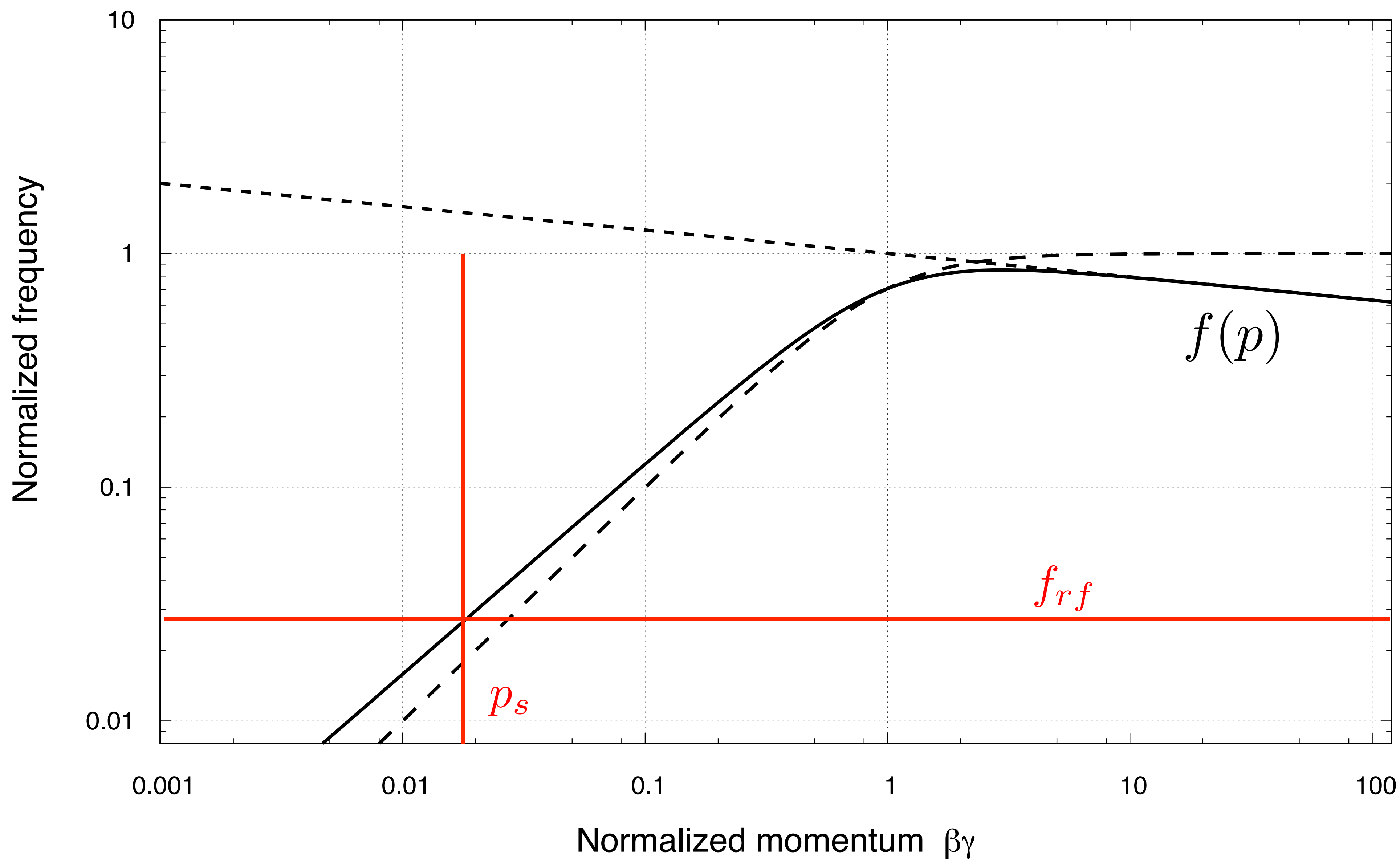

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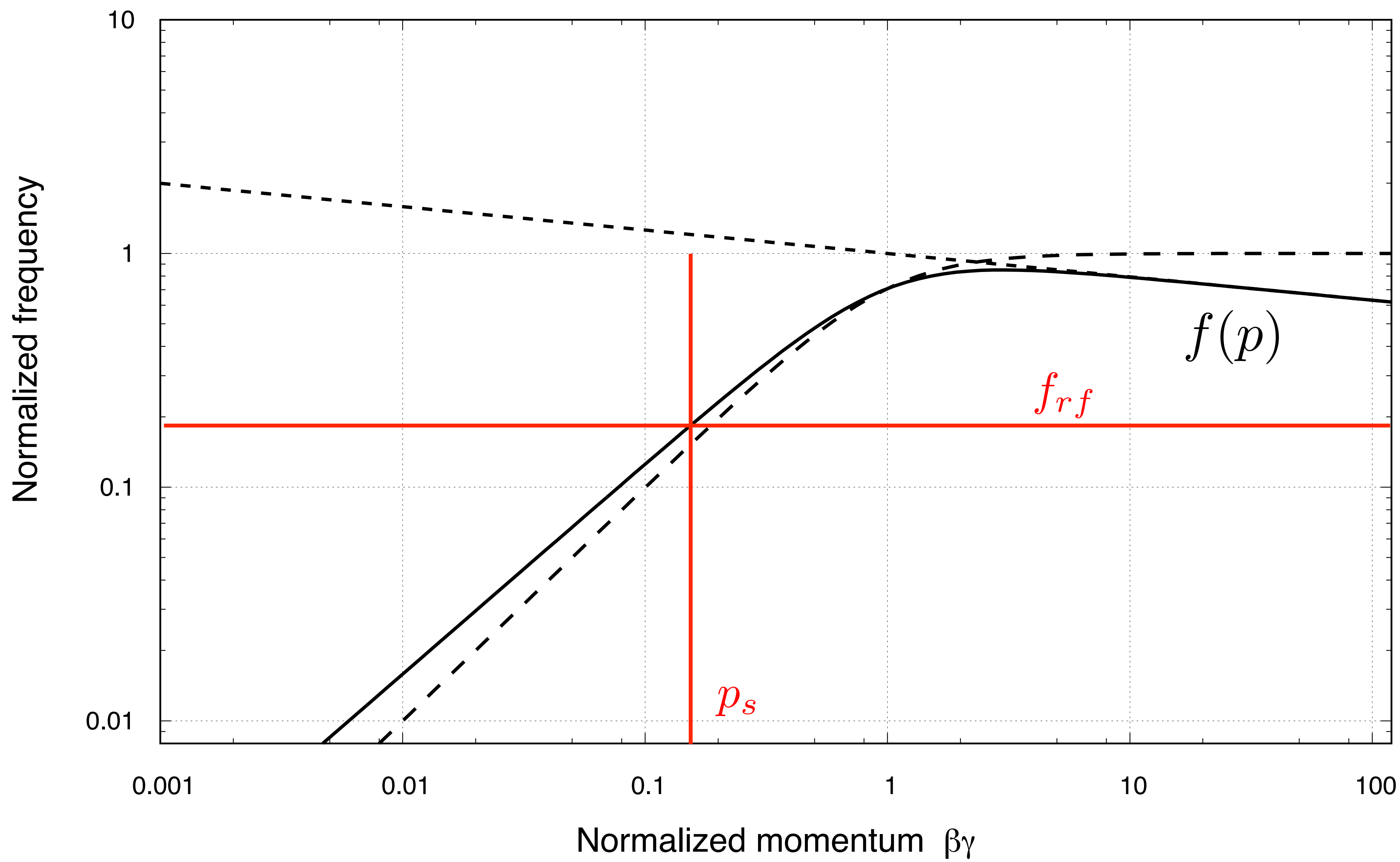
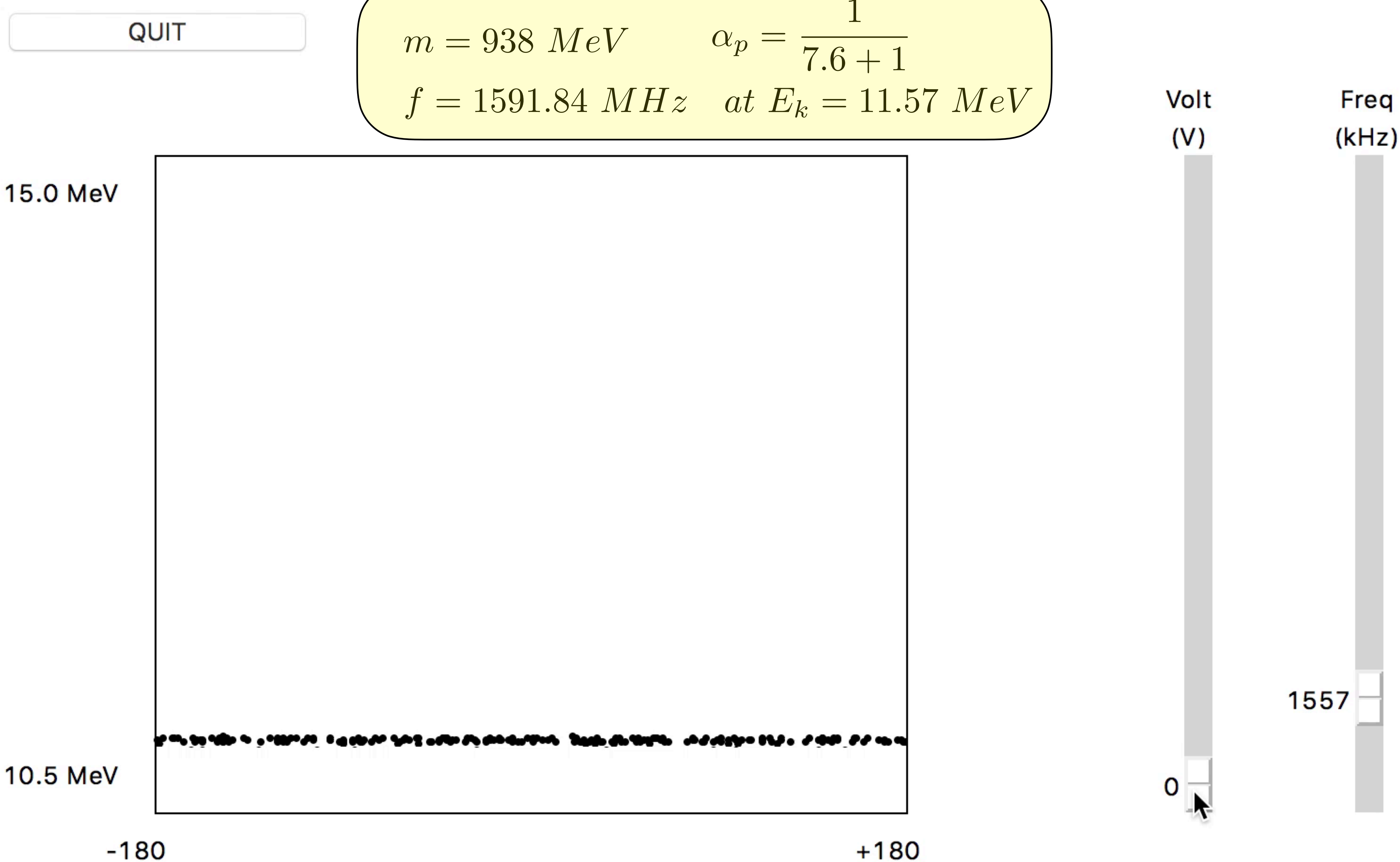


Demonstration 3: Rf acceleration

$$m = 938\ MeV \qquad \alpha_p = \frac{1}{7.6+1}$$

$$f = 1591.84\ MHz \quad at\ E_k = 11.57\ MeV$$



synchronous energy

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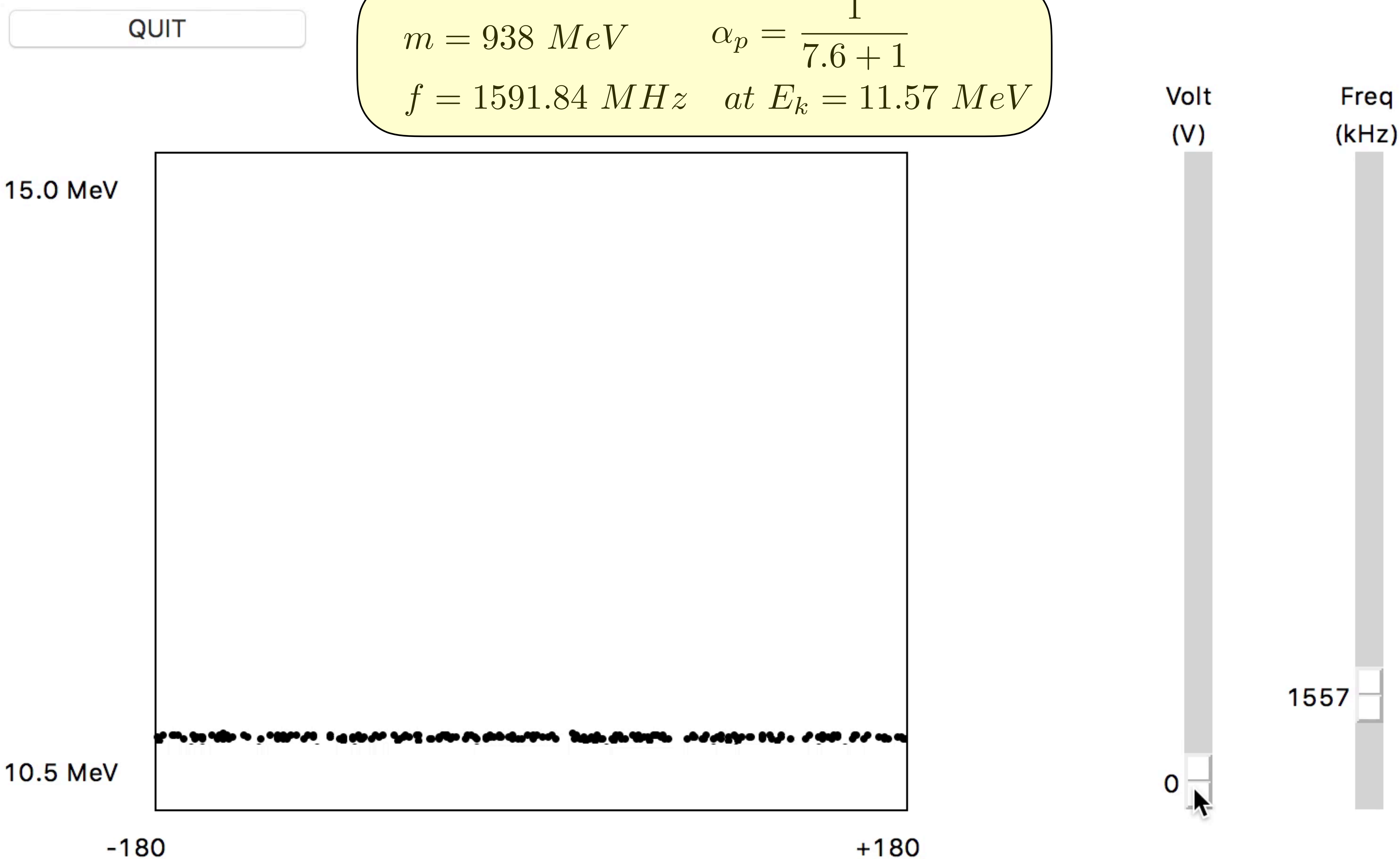

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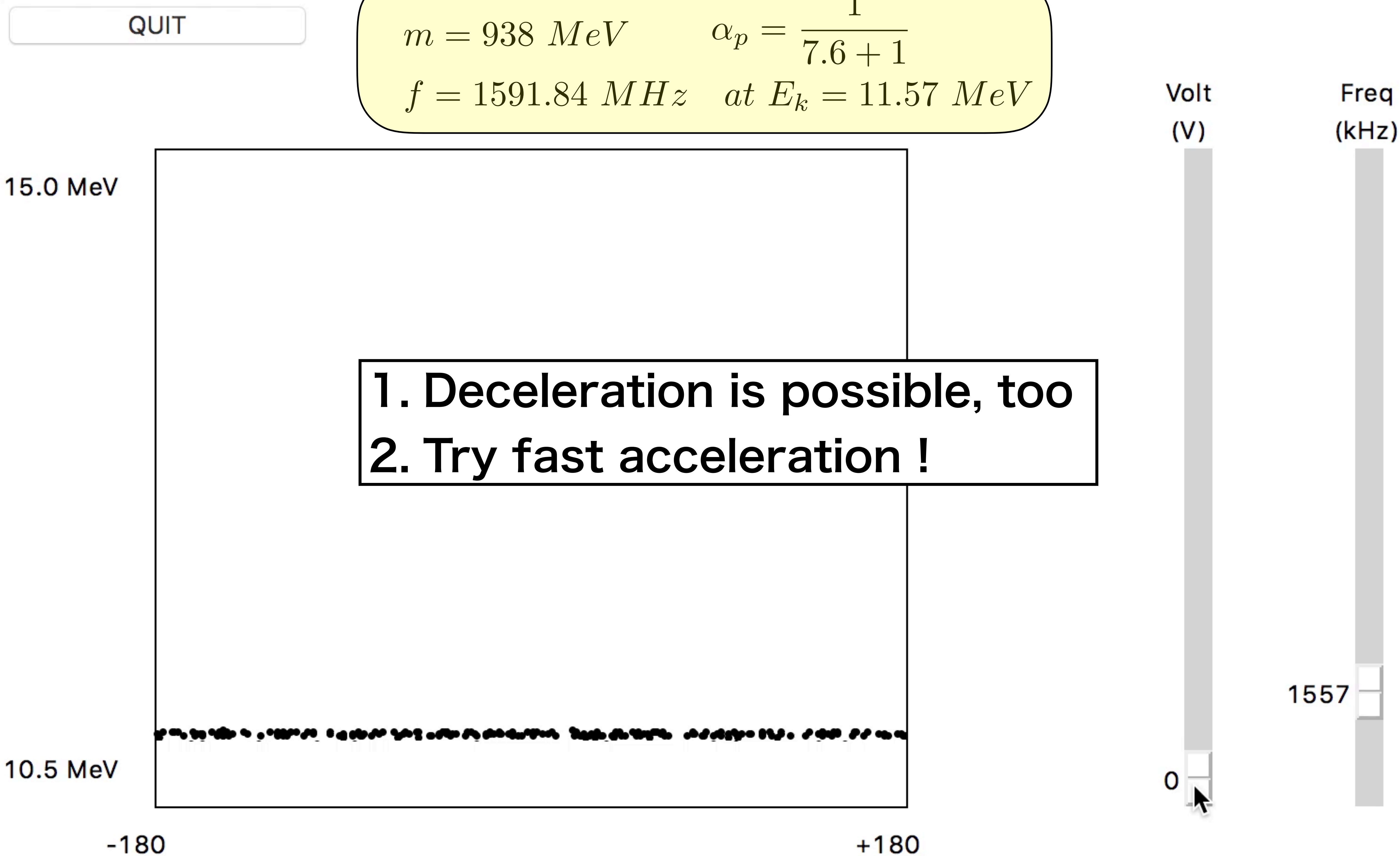
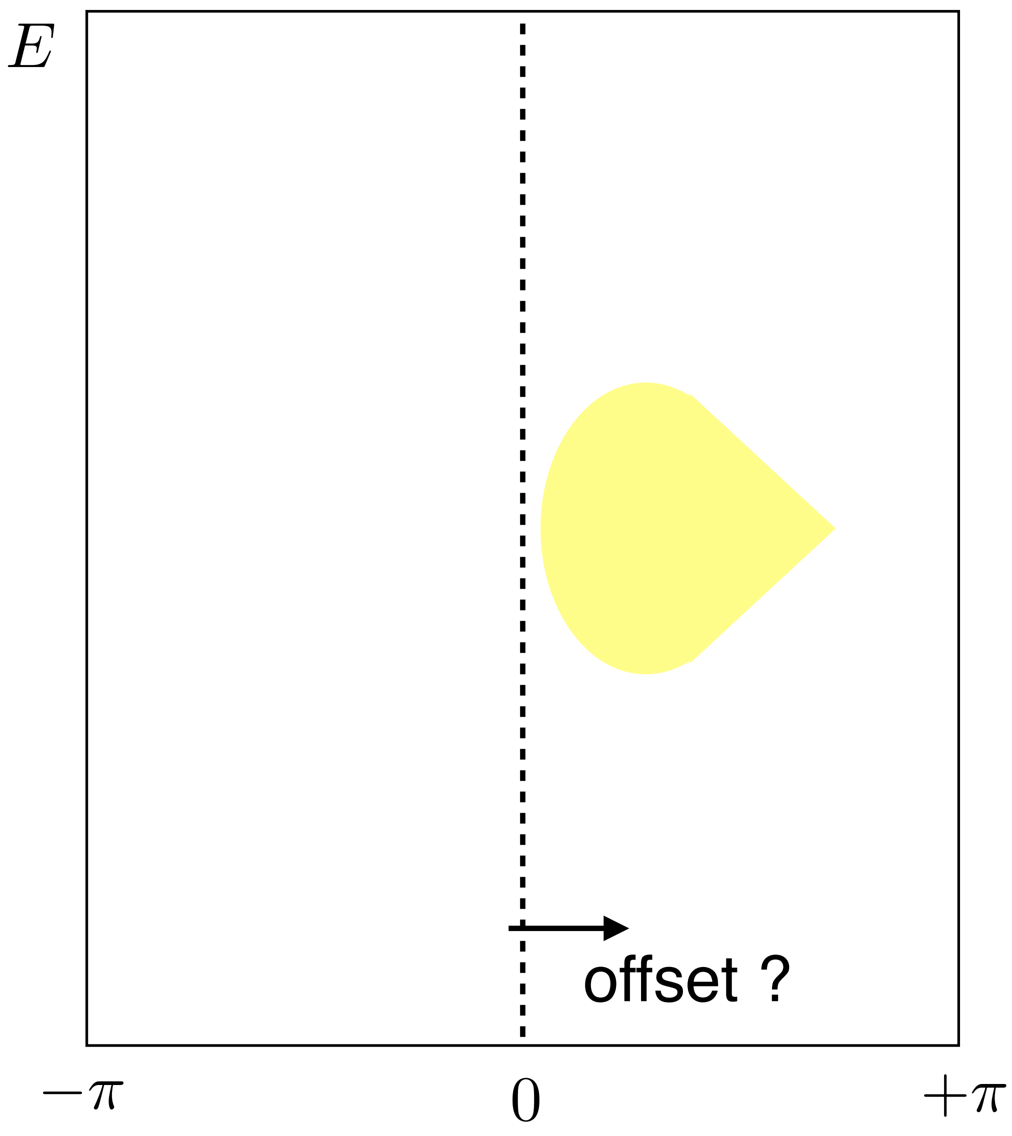


1. Deceleration is possible, too
2. Try fast acceleration !

synchronous energy

Synchronous particle



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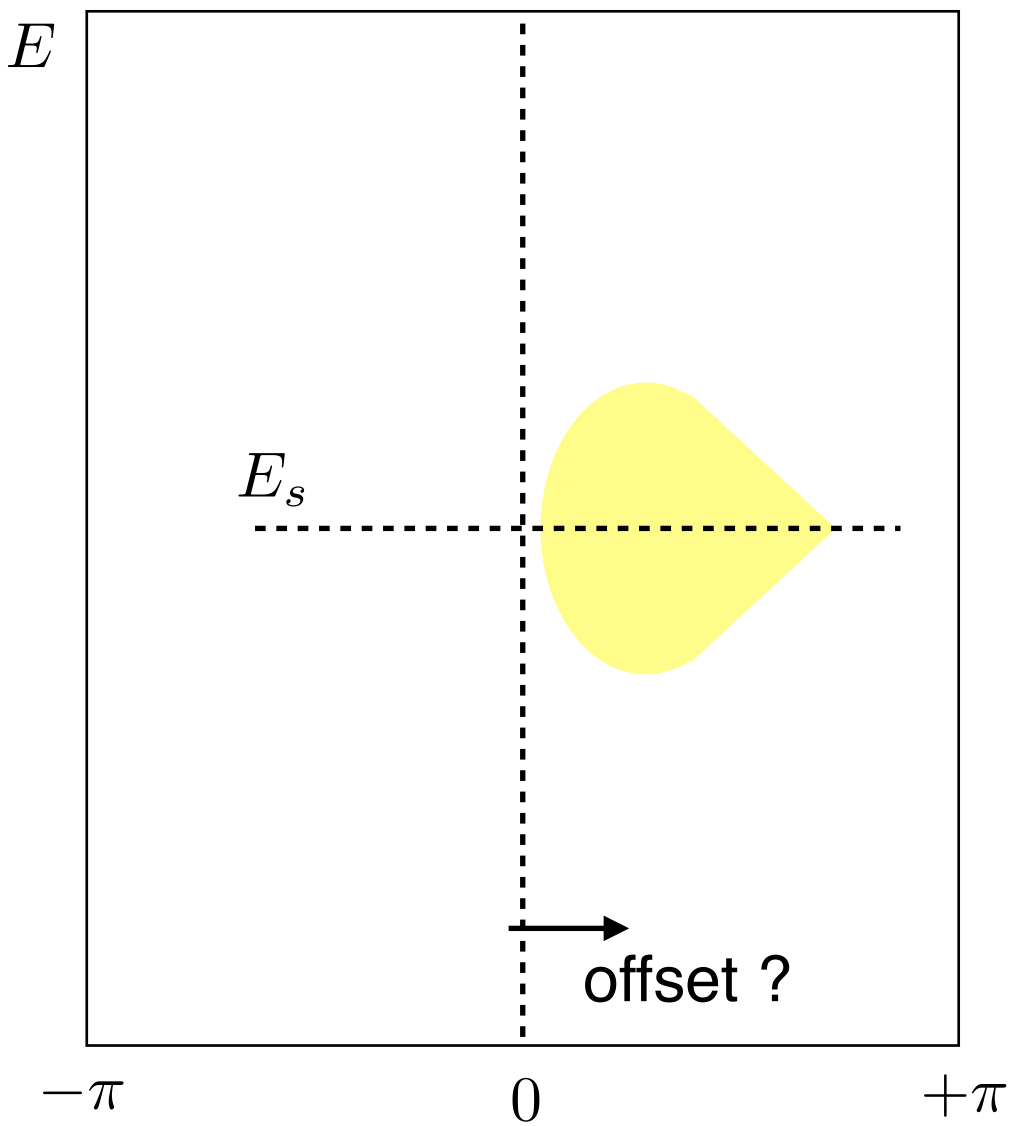
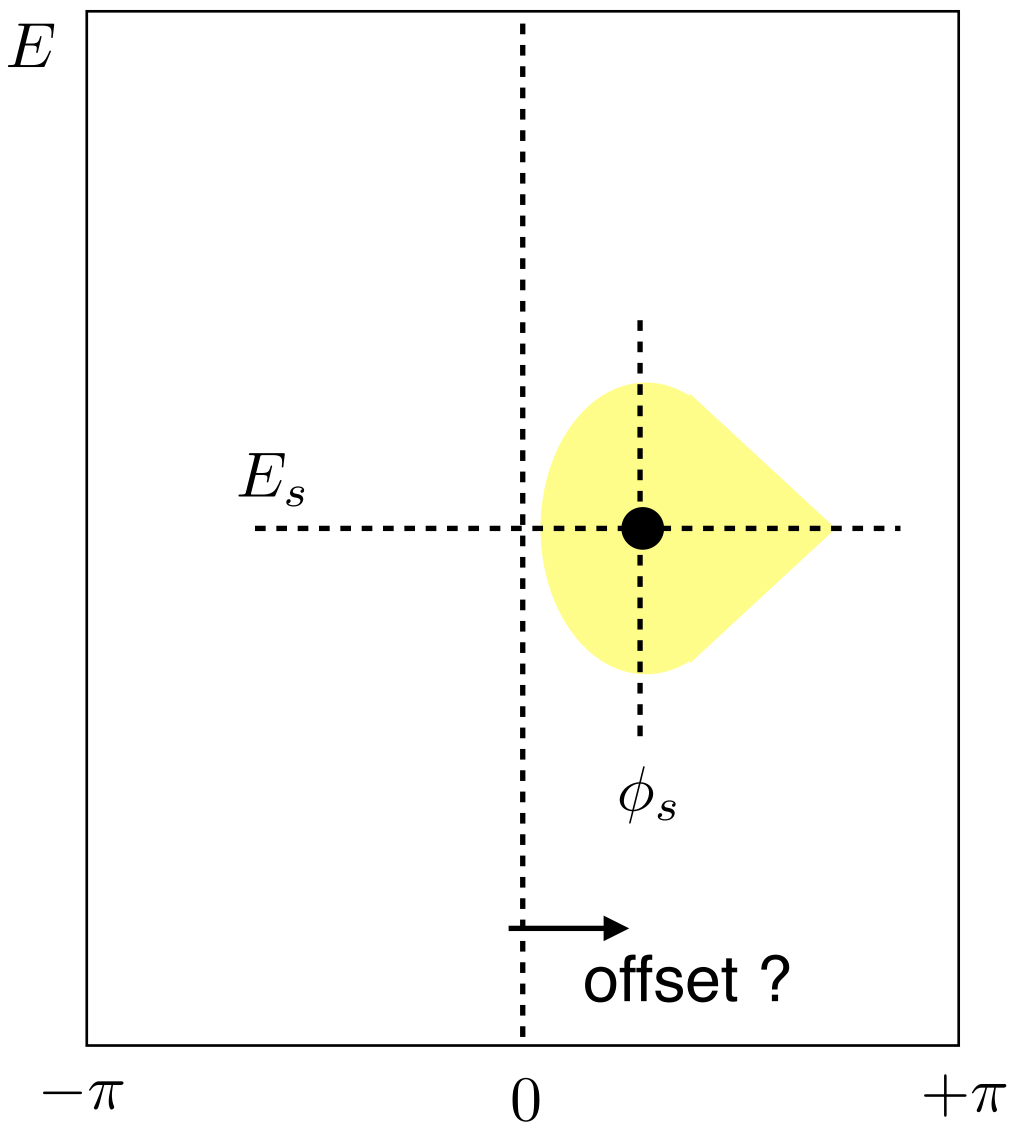


Rf frequency determines the *synchronous energy*.

$$h \cdot f(E_s) = f_{rf}$$

Synchronous particle



Rf frequency determines the *synchronous energy*.

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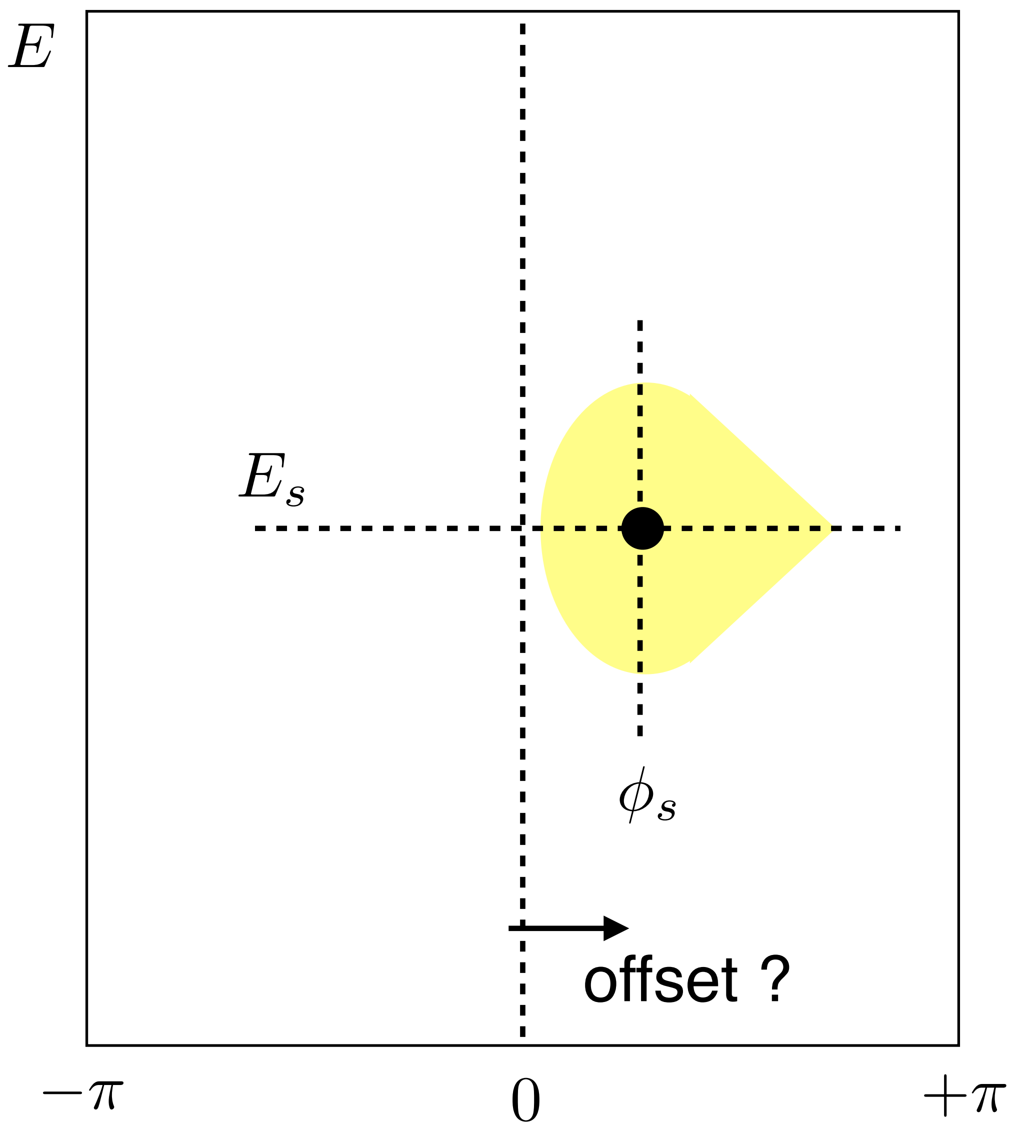
Synchronous phase is determined by the synchronous energy gain per turn,

$$qV \sin\phi_s = \frac{\mathrm{d}E_s}{\mathrm{d}N} = \frac{1}{f}\frac{\mathrm{d}E_s}{\mathrm{d}t}$$

Synchronous particle



Rf frequency determines the *synchronous energy*.

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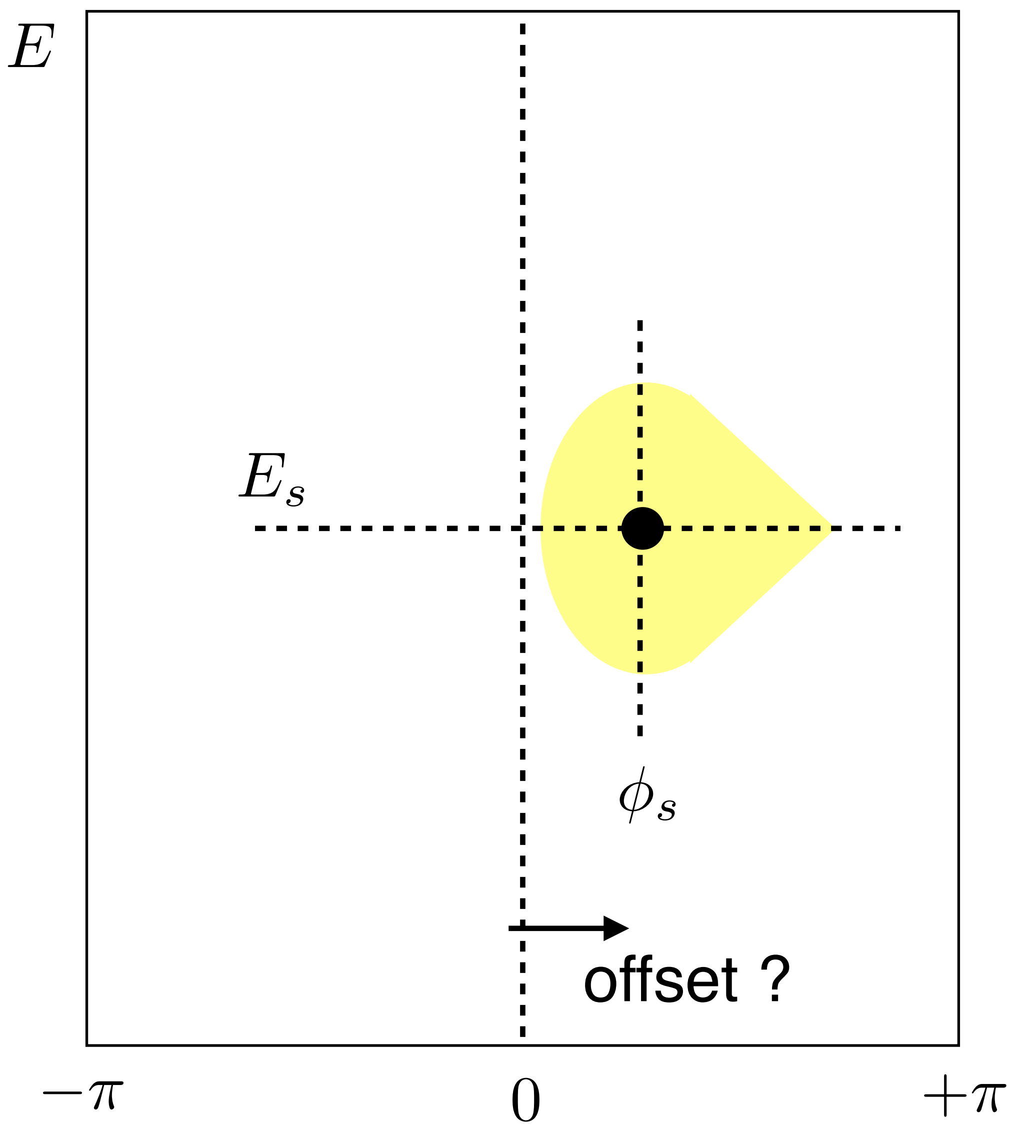
Synchronous phase is determined by the synchronous energy gain per turn, or, the ramping rate of rf frequency.

$$\begin{aligned} qV \sin\phi_s &= \frac{\mathrm{d}E_s}{\mathrm{d}N} = \frac{1}{f}\frac{\mathrm{d}E_s}{\mathrm{d}t} \\ &= \frac{1}{f_{rf}} \frac{1}{(\mathrm{d}f/\mathrm{d}E_s)} \frac{\mathrm{d}f_{rf}}{\mathrm{d}t} \end{aligned}$$

(ϕ_s, E_s) is a stable fixed point

Synchronous particle



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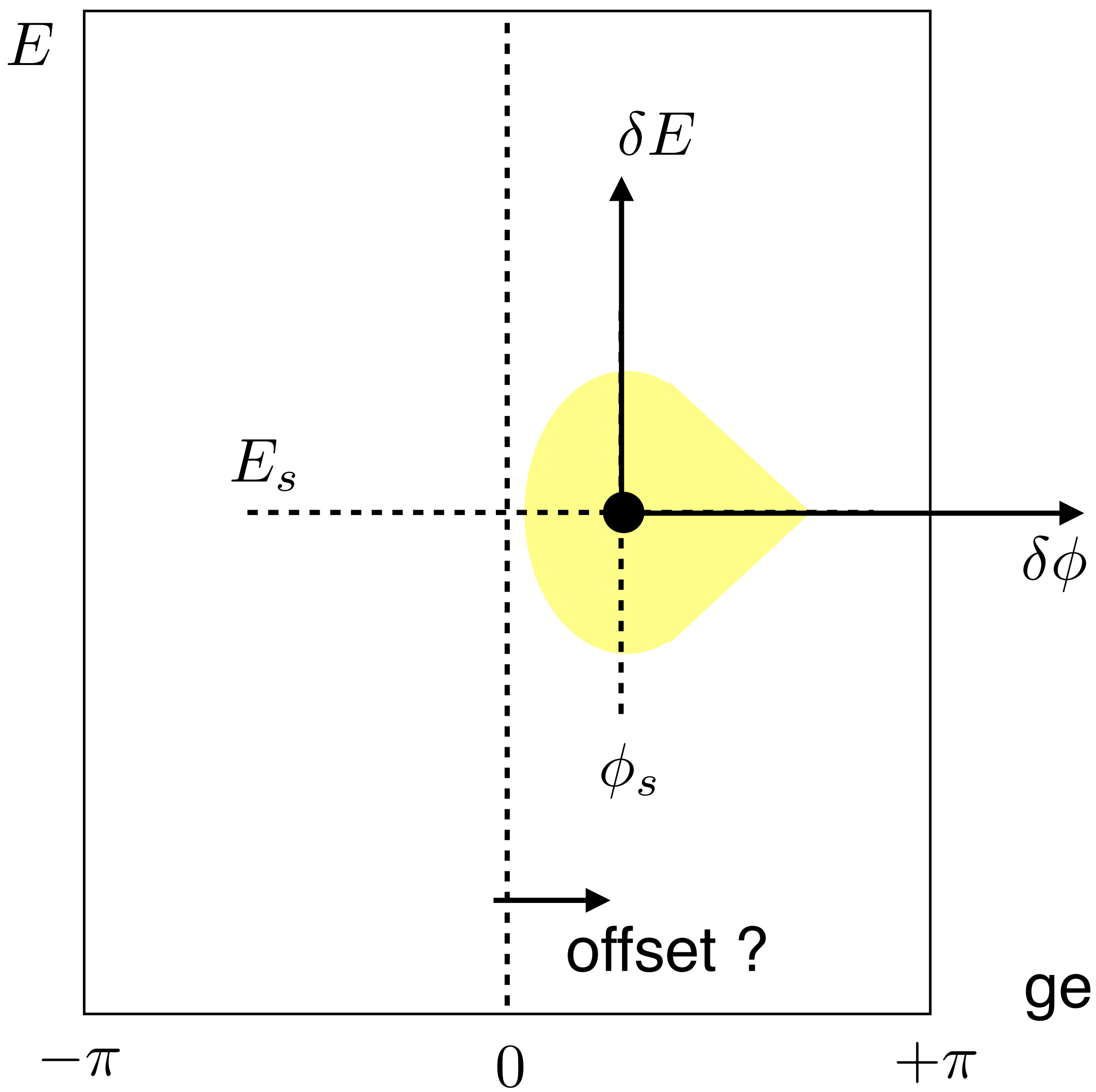
general particle

Synchronous particle



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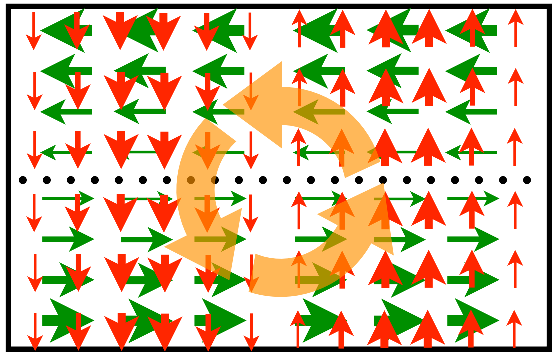
general particle

$$\begin{aligned} \phi &= \phi_s + \delta\phi \\ E &= E_s + \delta E \end{aligned}$$

Equation of motion with respect to $(\ \delta\phi\ ,\ \delta E\ )$ $\longrightarrow$

Quantitative analysis



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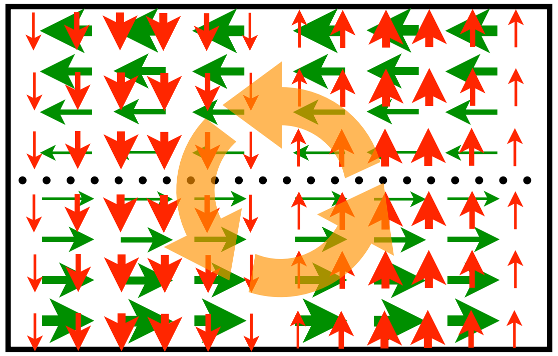
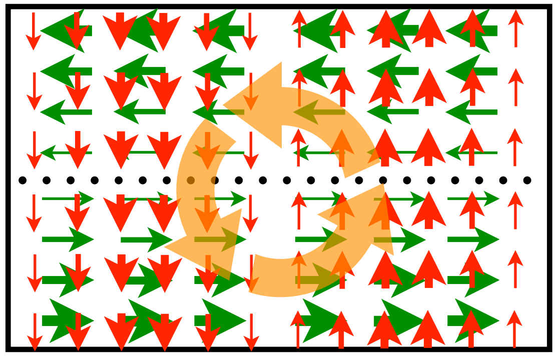


One turn phase advance

$$\Delta \frac{\phi}{2\pi h} = \eta \frac{\delta p}{p_s}$$

Quantitative analysis



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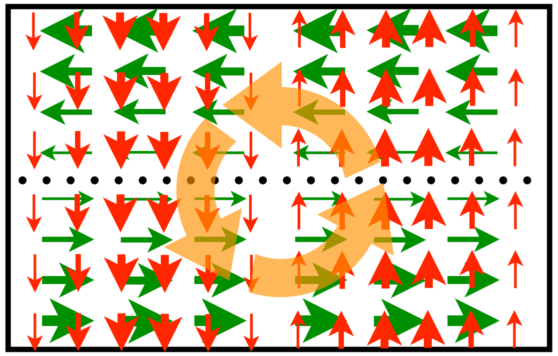

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One energy gain

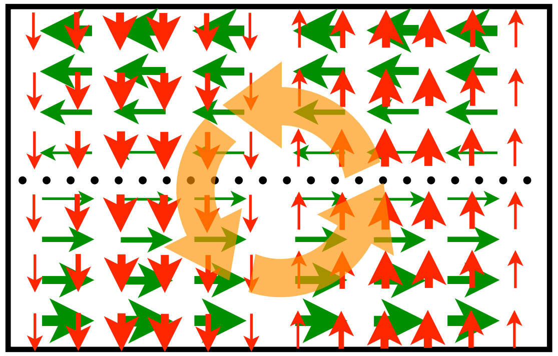
$$\Delta \left(E_s + \delta E\right) \qquad\qquad = qV \sin\phi$$

Quantitative analysis

$$E = E_s + \delta E$$

$$\Delta \longleftrightarrow \frac{\mathrm{d}}{\mathrm{d}N}$$

Smooth approximation



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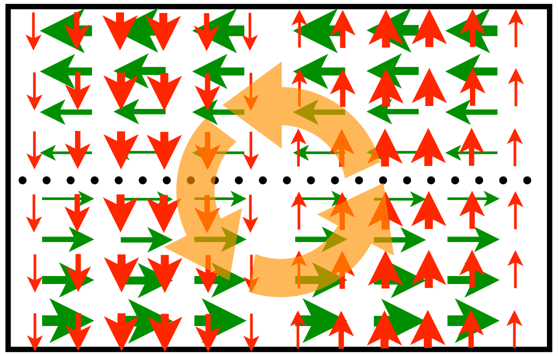

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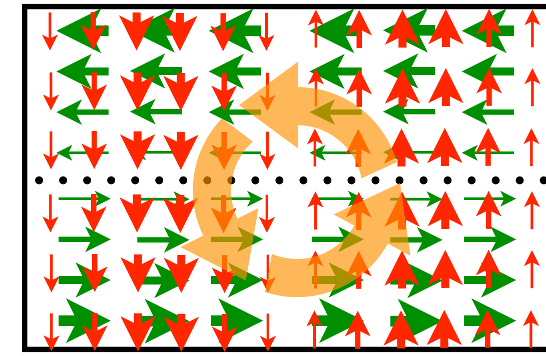

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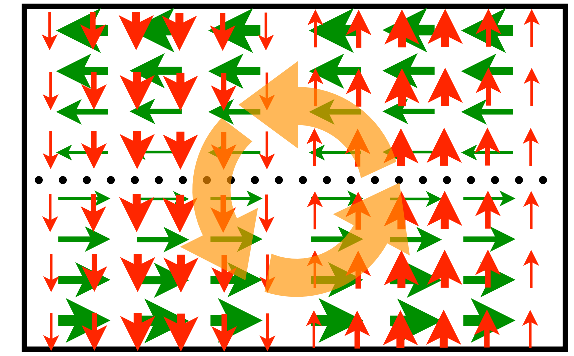
$$\boxed{H(\phi, \delta E; N) = \frac{1}{2} \frac{2\pi h \eta}{\beta_s^2 E_s} \left(\delta E\right)^2 + qV \left(\cos\phi_s + \sin\phi_s \cdot \phi\right)}$$

Phase trajectory in $(\phi, \delta E)$ space $\longrightarrow$ Next page

Quantitative analysis



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Frequency of small amplitude oscillation at $\phi \simeq \phi_s$.

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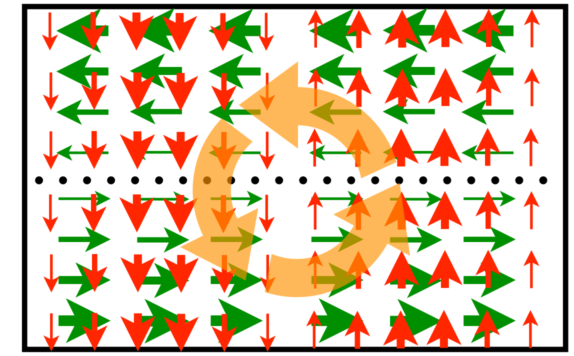

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Phase trajectory in $(\phi, \delta E)$ space $\longrightarrow$ Next page

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$$\frac{\mathrm{d}^2}{\mathrm{d}N^2} \delta\phi = \left(2\pi \frac{f_s}{f}\right)^2 \delta\phi \qquad \frac{f_s}{f} = \sqrt{-\frac{h\eta}{2\pi} \frac{qV}{\beta_s^2 E_s} \cos\phi_s}$$

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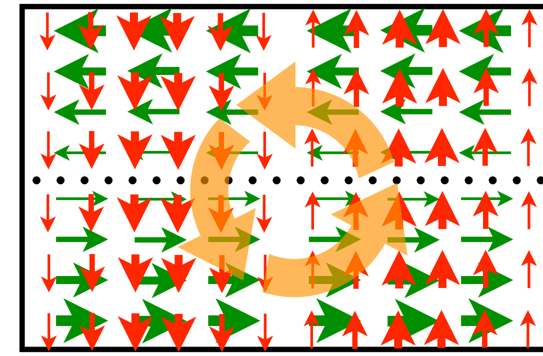

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= synchrotron tune

Phase trajectory

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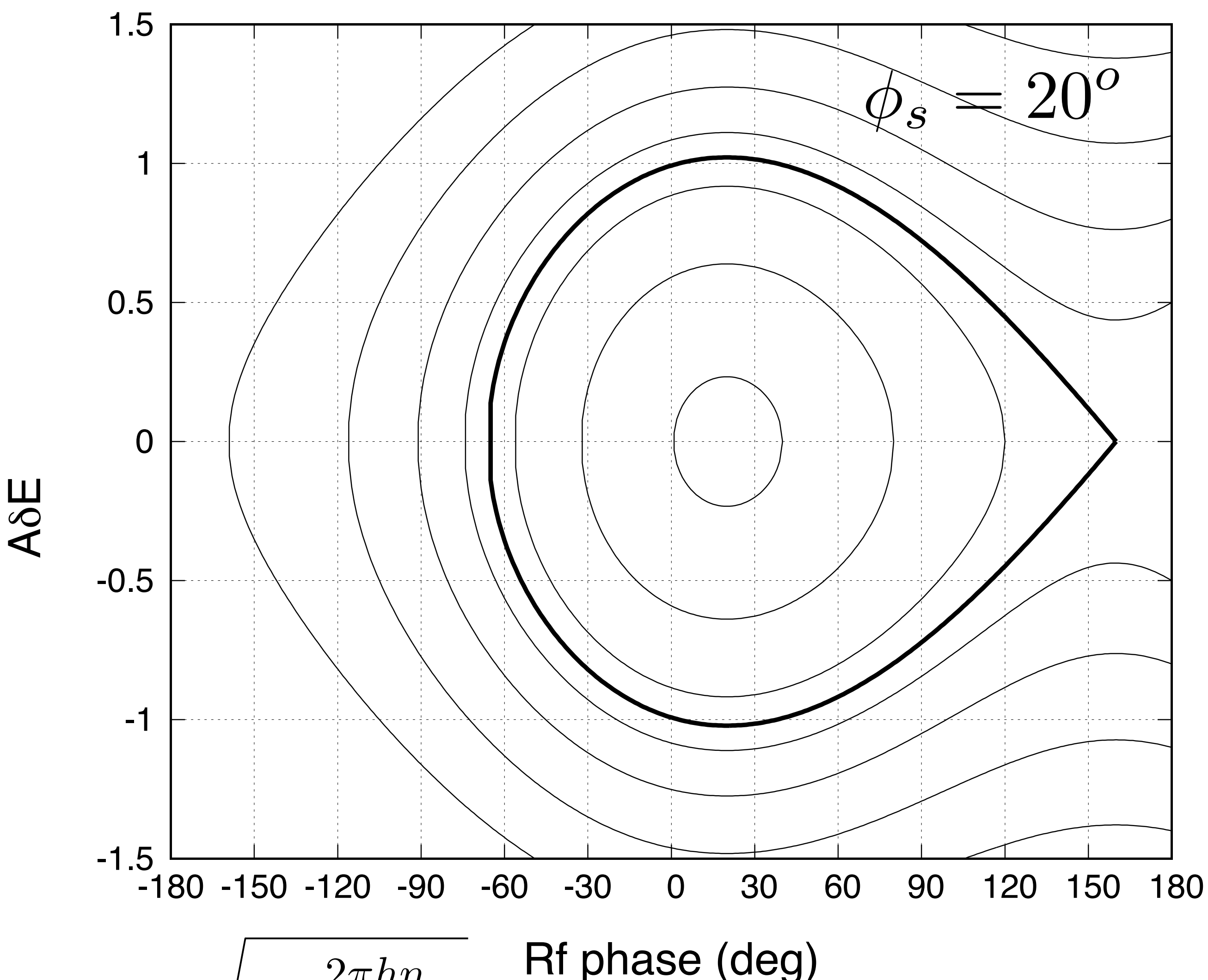

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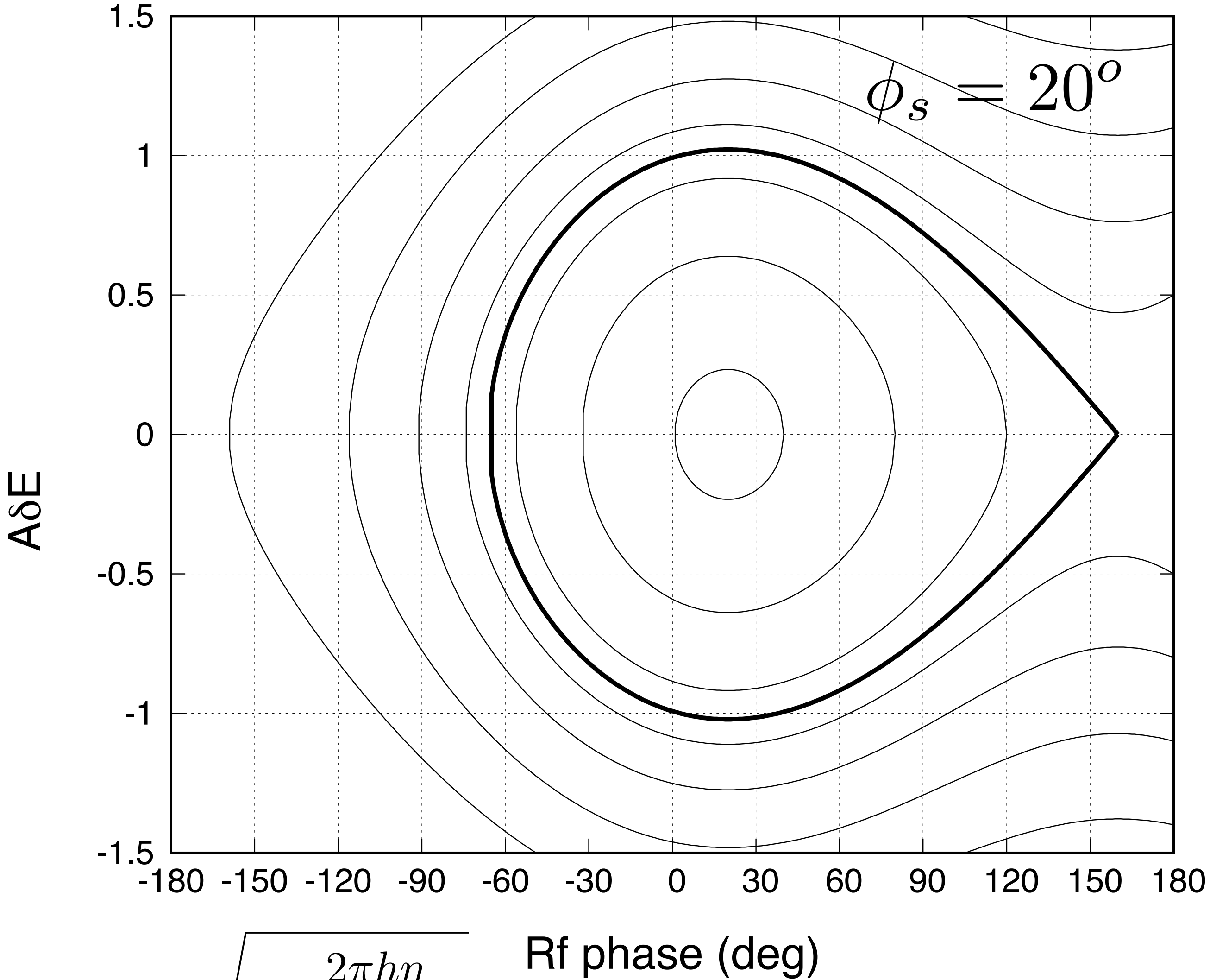

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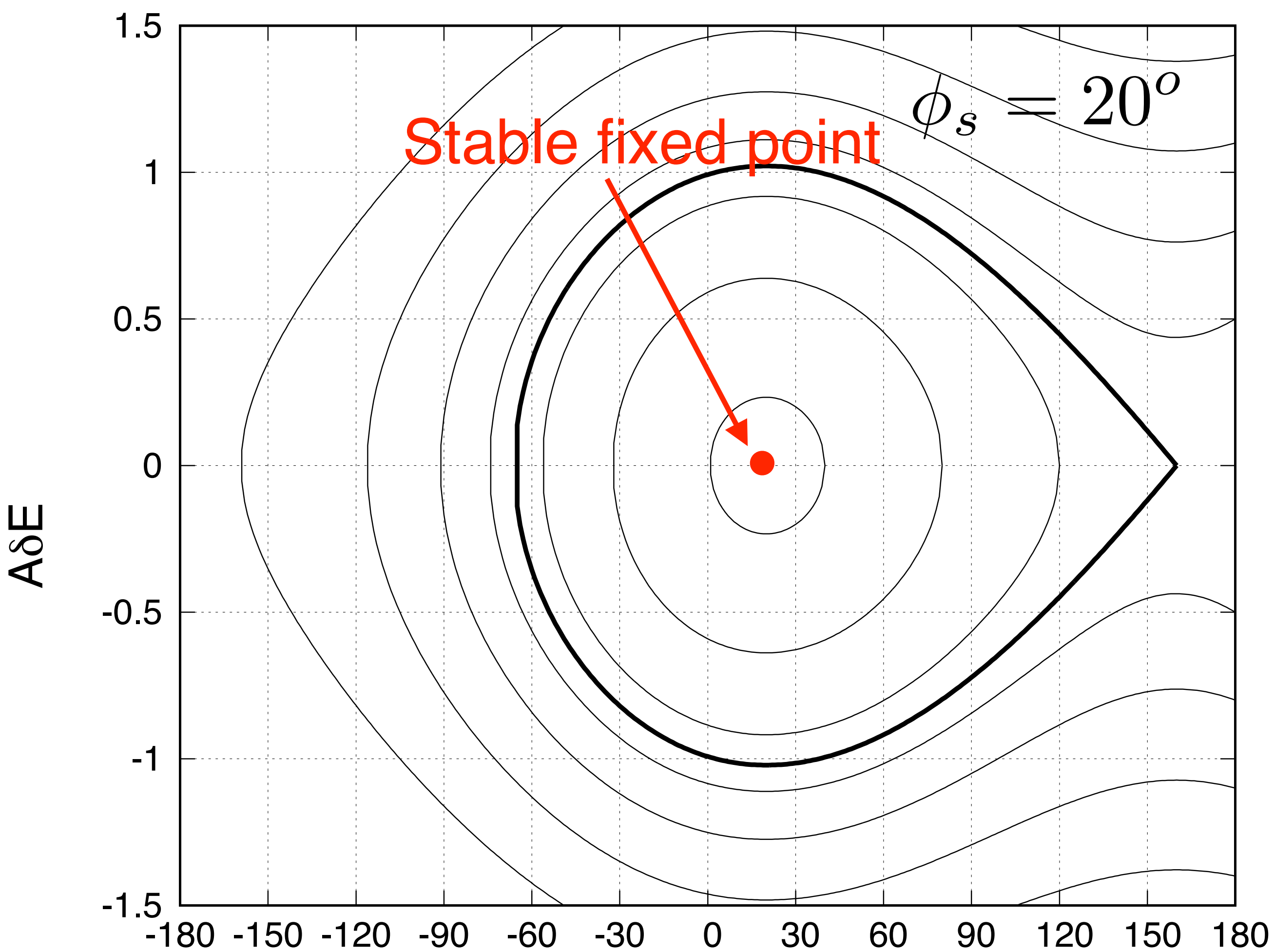

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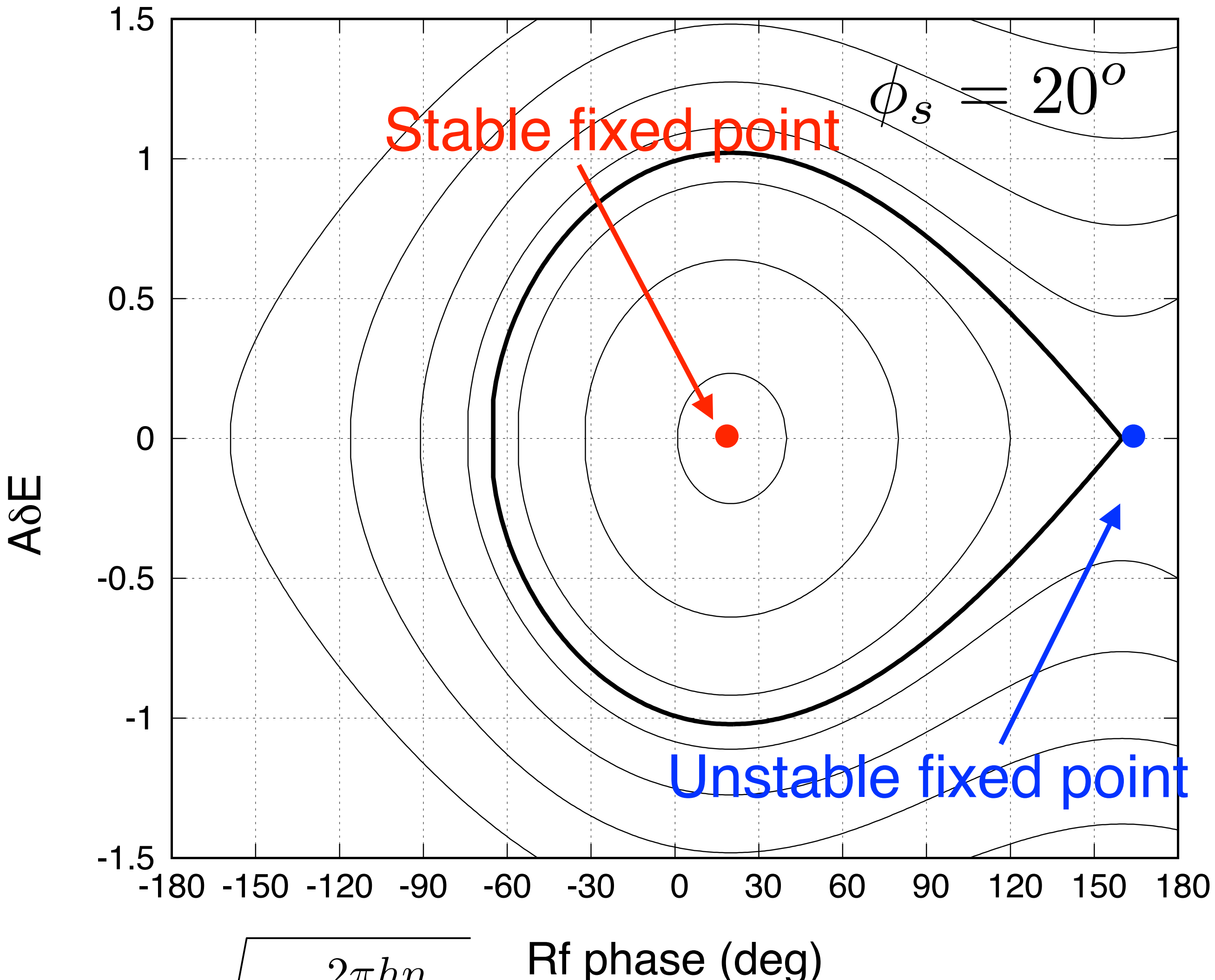

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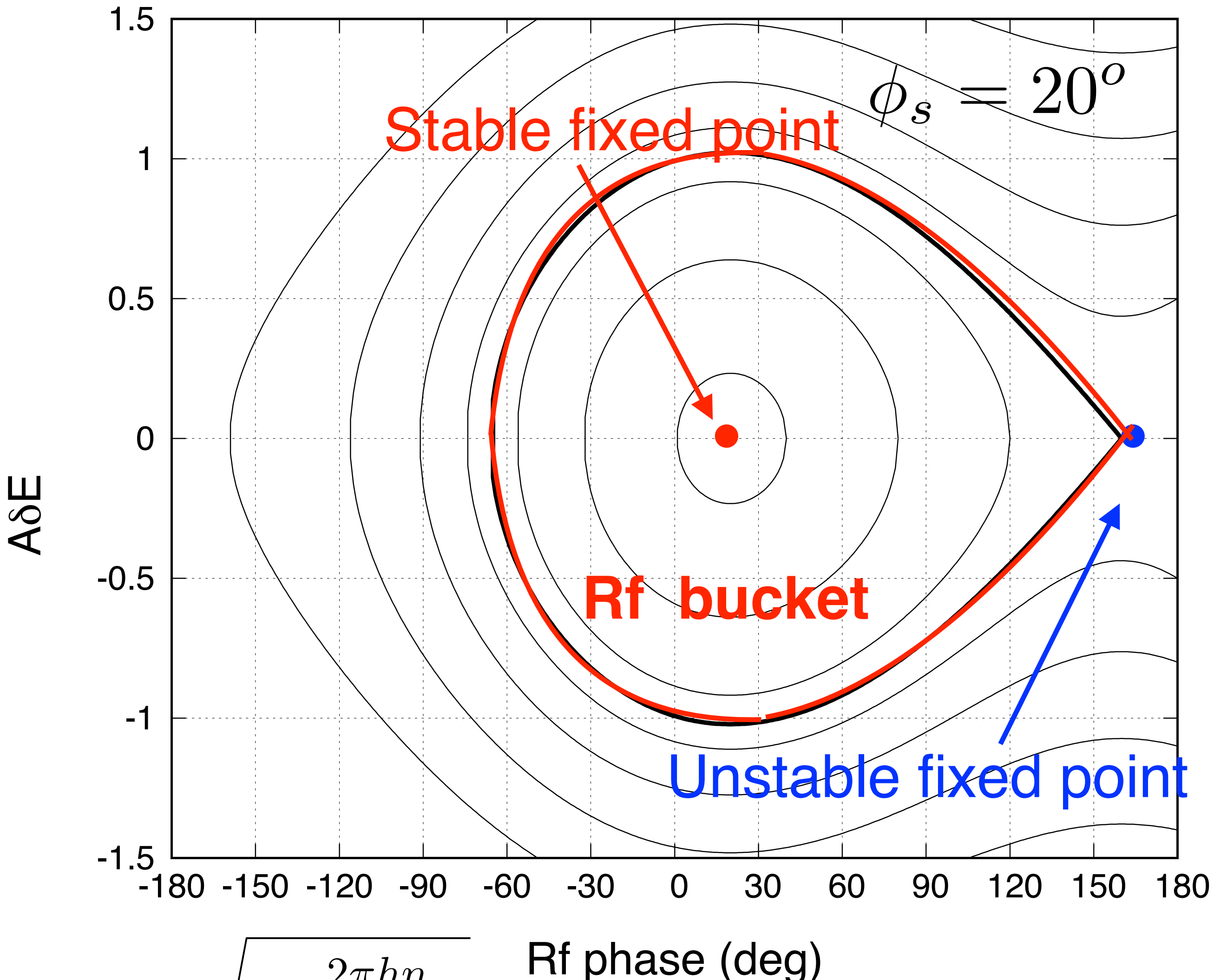

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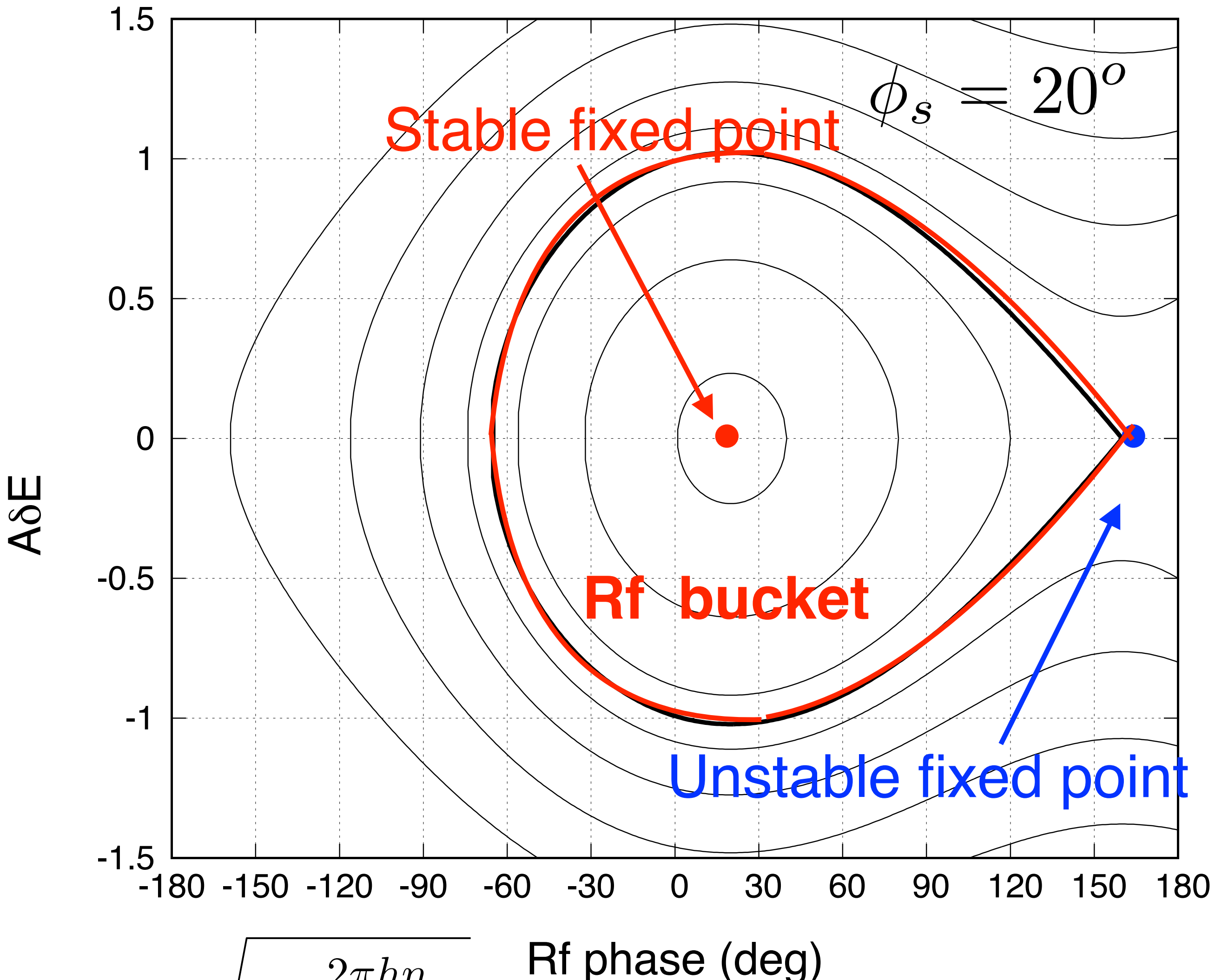

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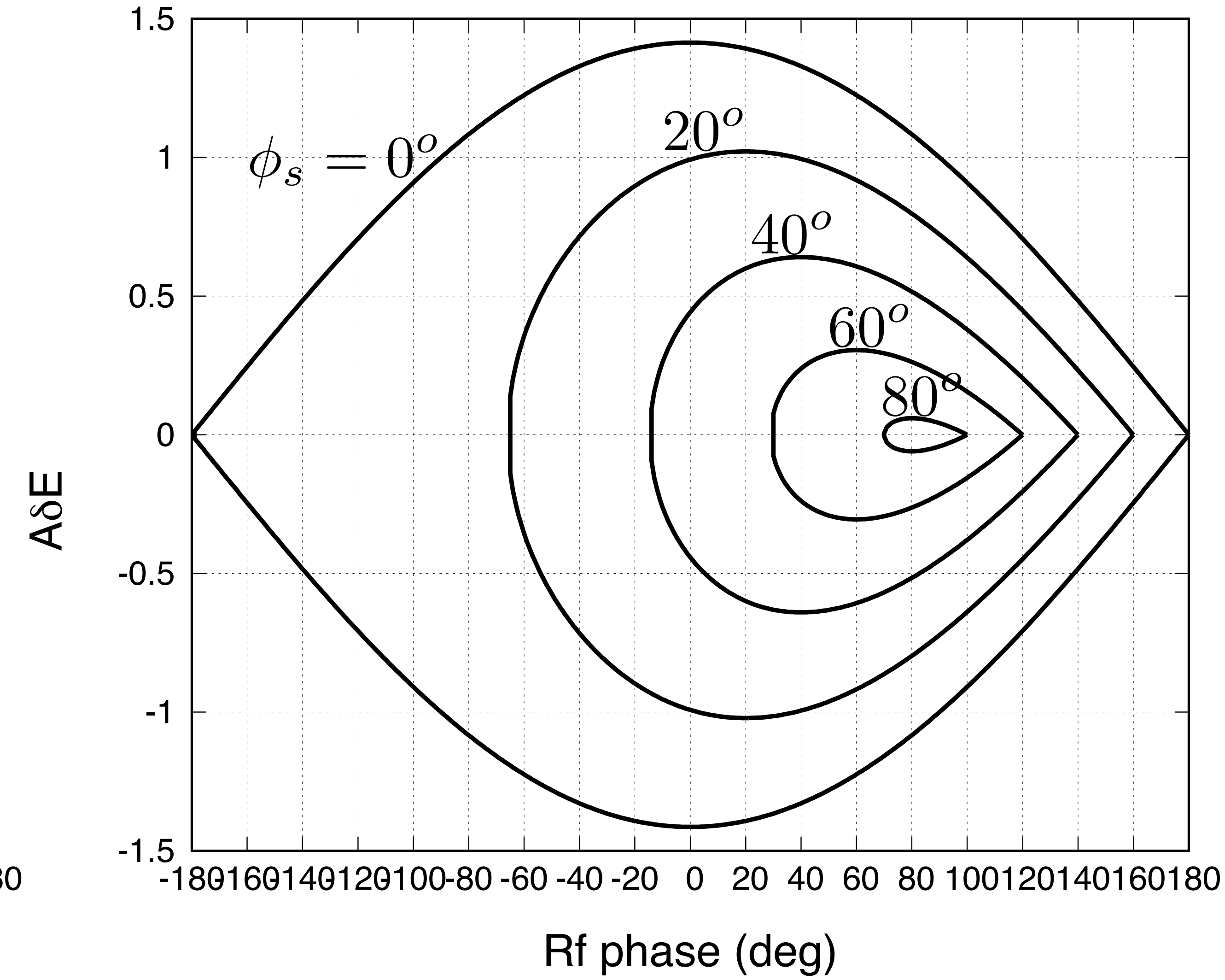

TOPICS AROUND RF ACCELERATION IN A FFAG

keywords

Stationary bucket,
Harmonic number jump,
Rf stacking

Characteristics of FFAG

Free from magnetic field ramping

Huge momentum aperture

Characteristics of FFAG

Free from magnetic field ramping

Huge momentum aperture

Characteristics of FFAG

Free from magnetic field ramping

Huge momentum aperture

Characteristic

Fast acceleration

Characteristics of FFAG

Free from magnetic field ramping

Huge momentum aperture

Characteristic aiming to

Fast acceleration High repetition rate

Characteristics of FFAG

Free from magnetic field ramping

Huge momentum aperture

Characteristic	aiming to
Fast acceleration	High repetition rate
Fixed frequency rf	Continuous acceleration

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Characteristic	aiming to	
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		Stacking at higher energy orbit

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	High pulse intensity	Stacking at higher energy orbit

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Fast acceleration

Fixed frequency rf

FFAG accelerator is free from magnetic field ramping,
but

still not capable of continuous acceleration
if rf frequency is varied.

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Is it possible to accelerate beams with a fixed frequency rf ?
Aiming a continuous acceleration

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Bucket height $\delta E \propto \sqrt{\frac{\beta^2}{\pi h \eta} EqV}$

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$\downarrow$

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Bucket height $\delta E \propto \sqrt{\frac{\beta^2}{\pi h \eta} E q V}$ is higher if

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(2) Rf voltage is higher
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η is low

Near transition energy

Vertical FFAG at ultra-relativistic energy

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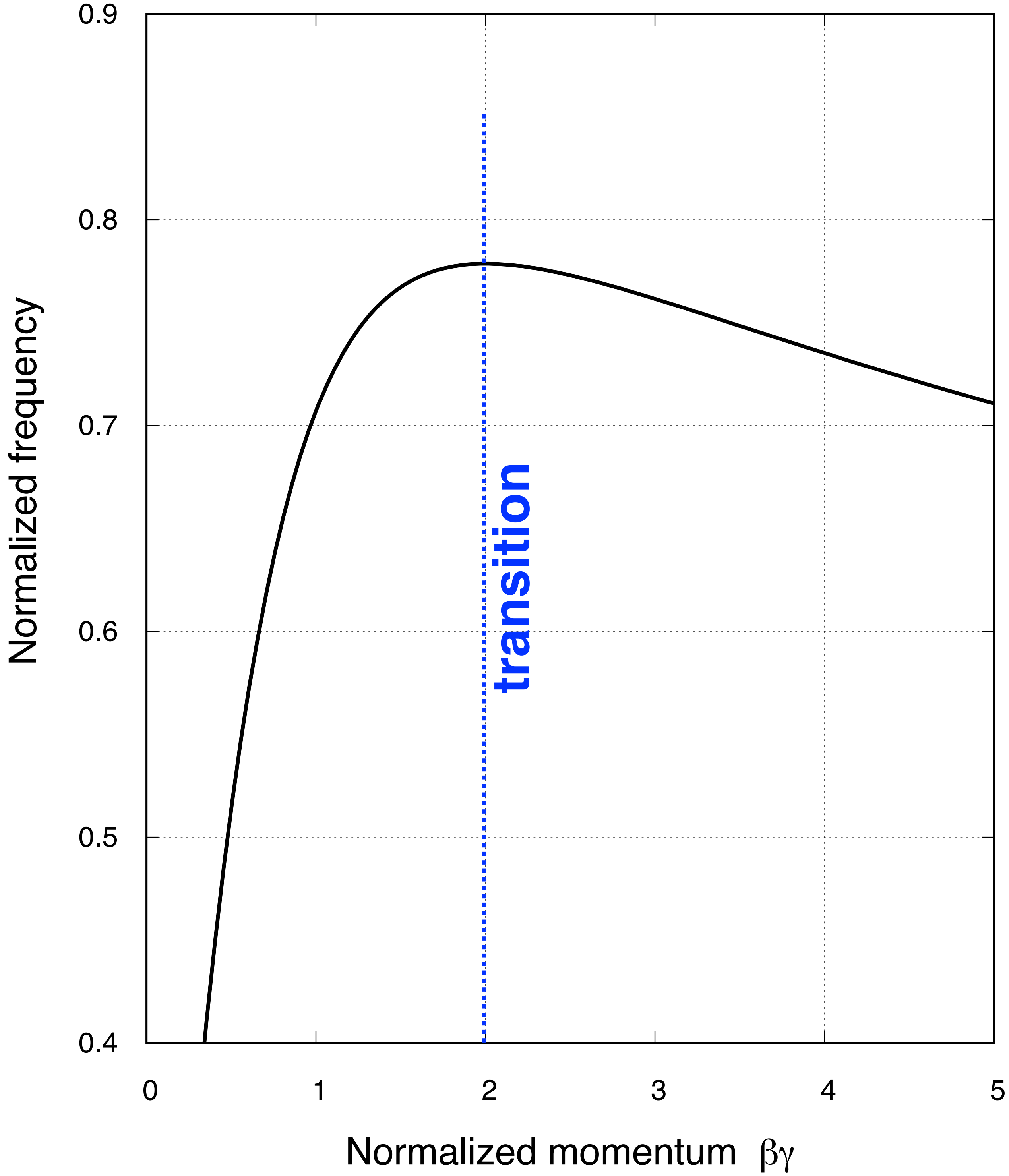
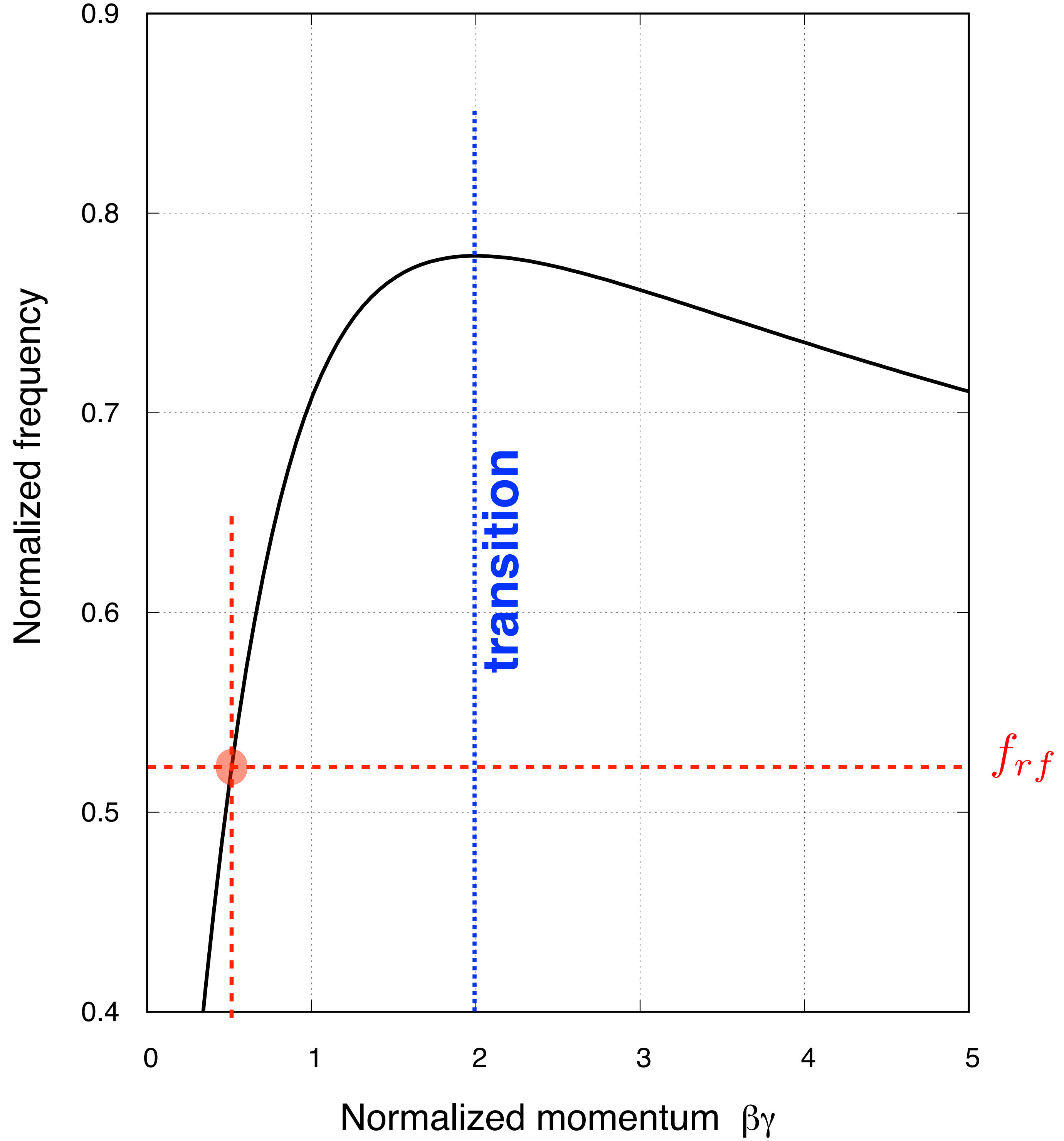


Near the transition



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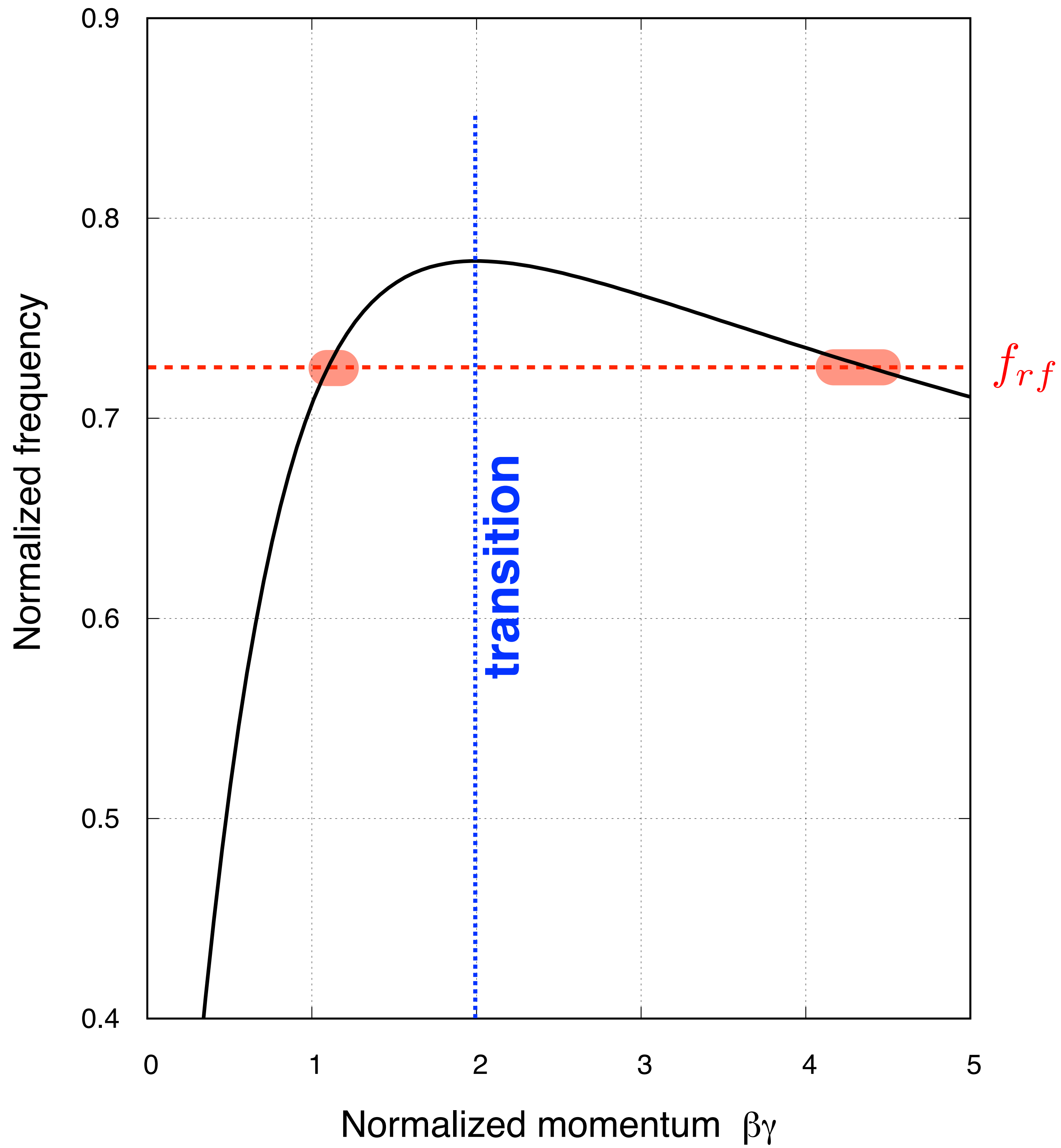

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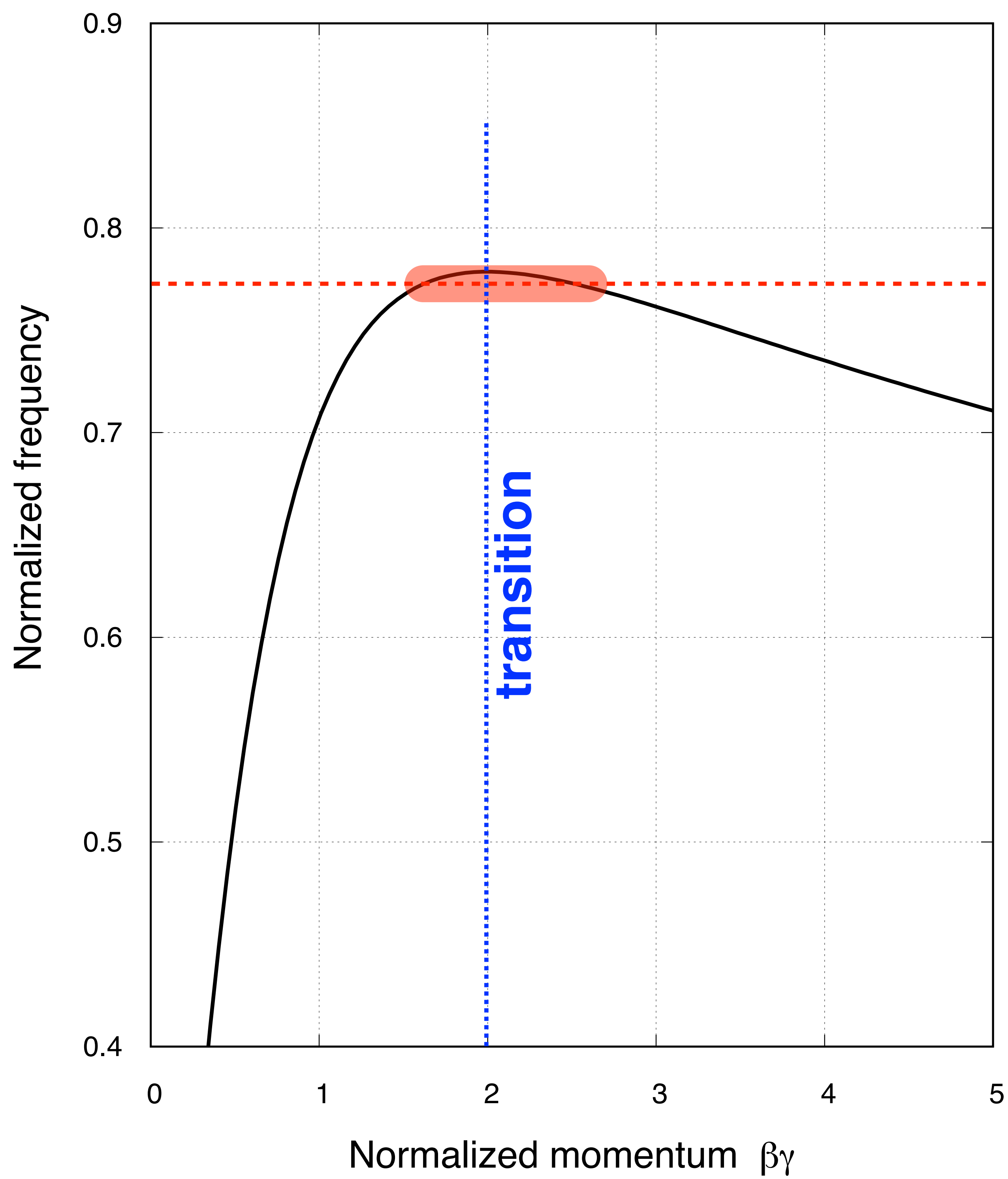

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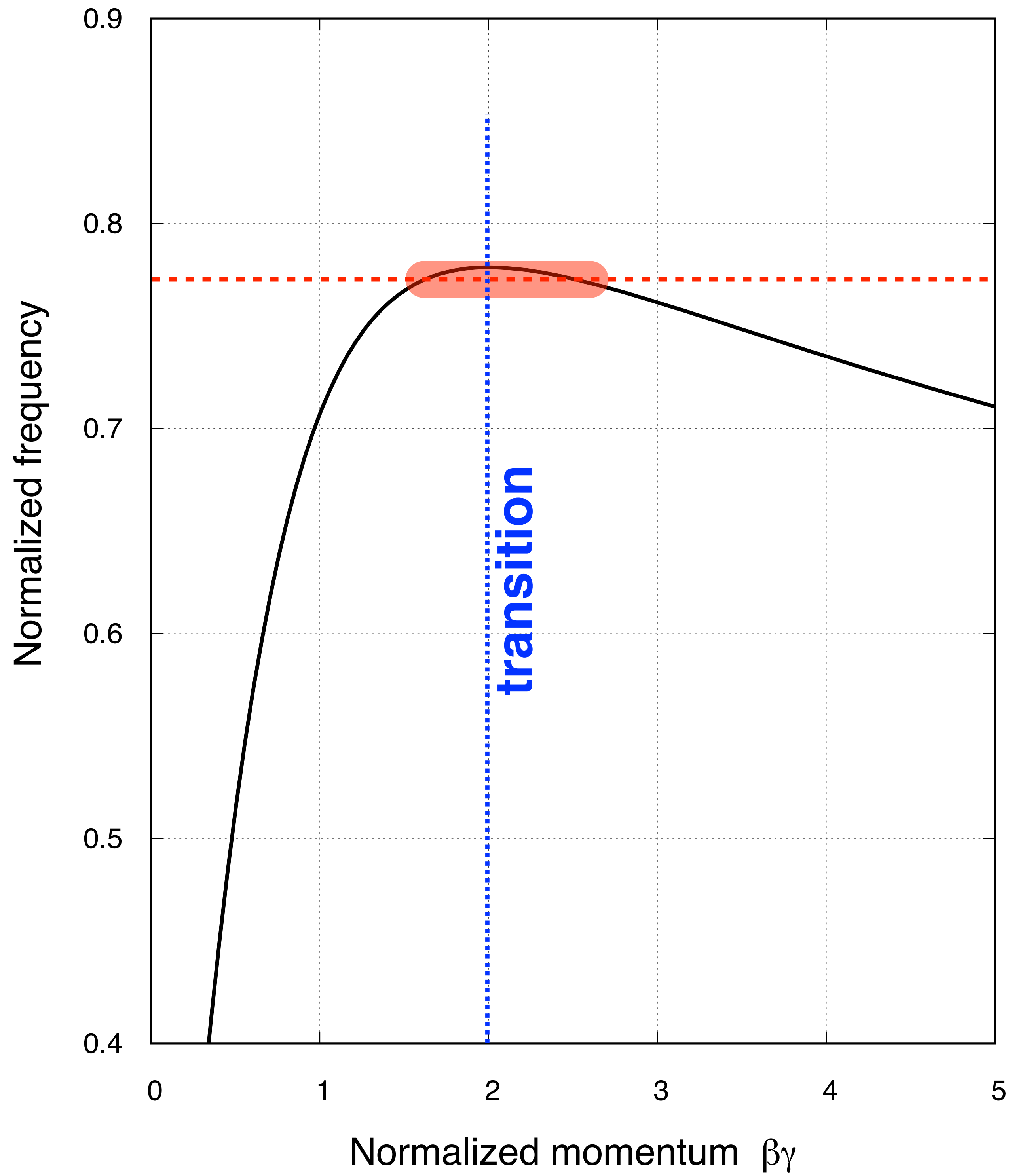
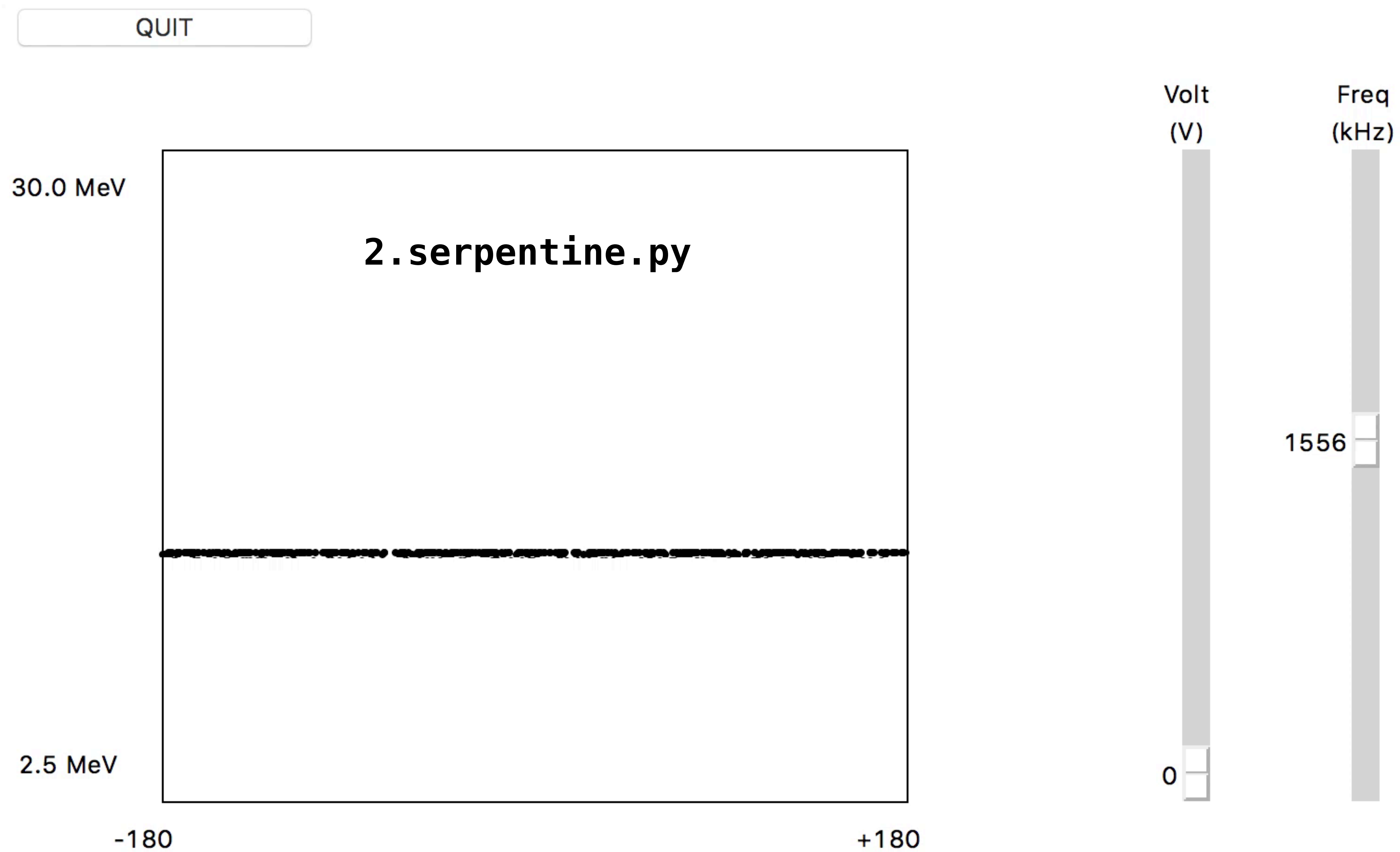


Extreme case;
Serpentine acceleration (next)

Demonstration 4: Serpentine acc.



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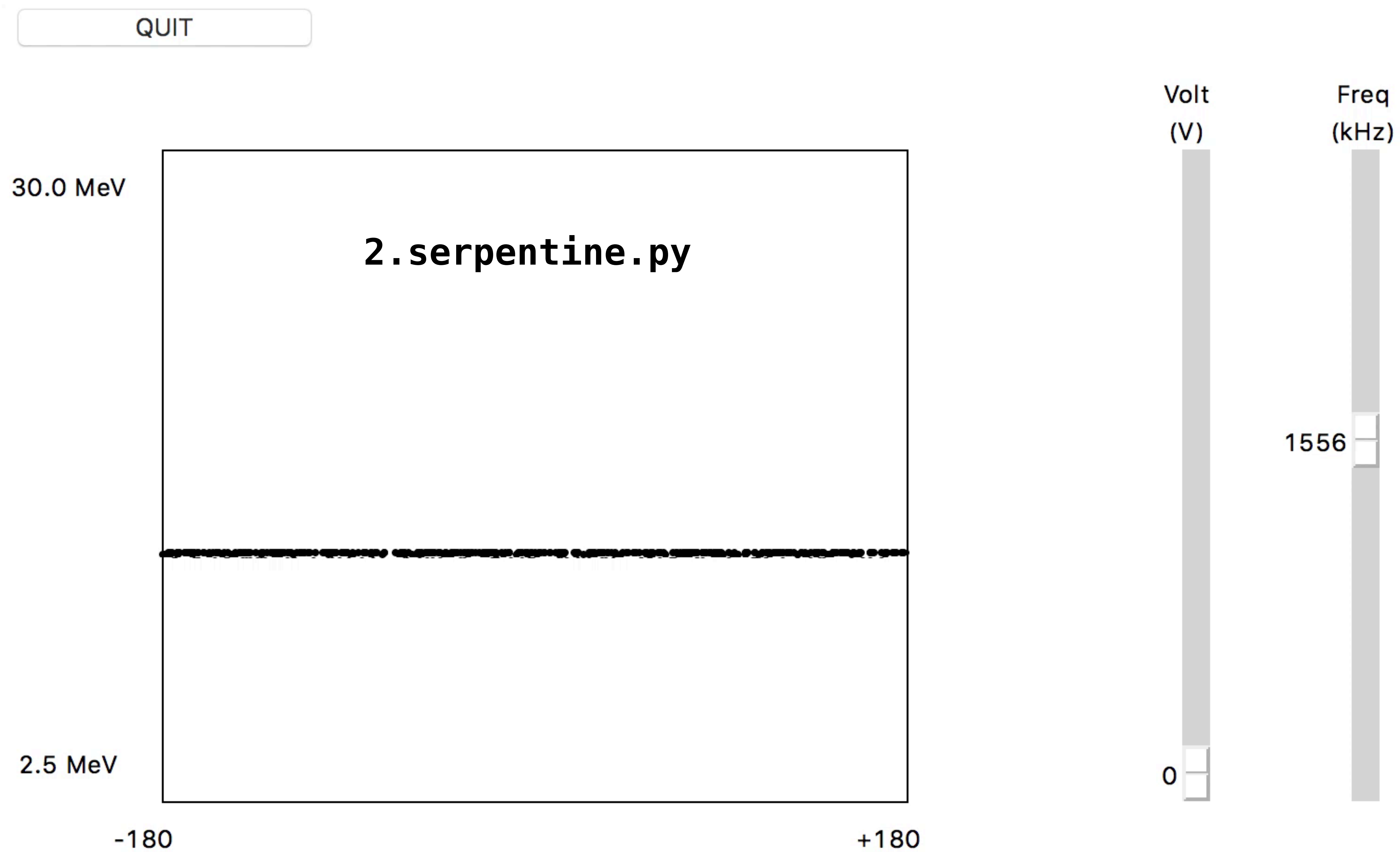
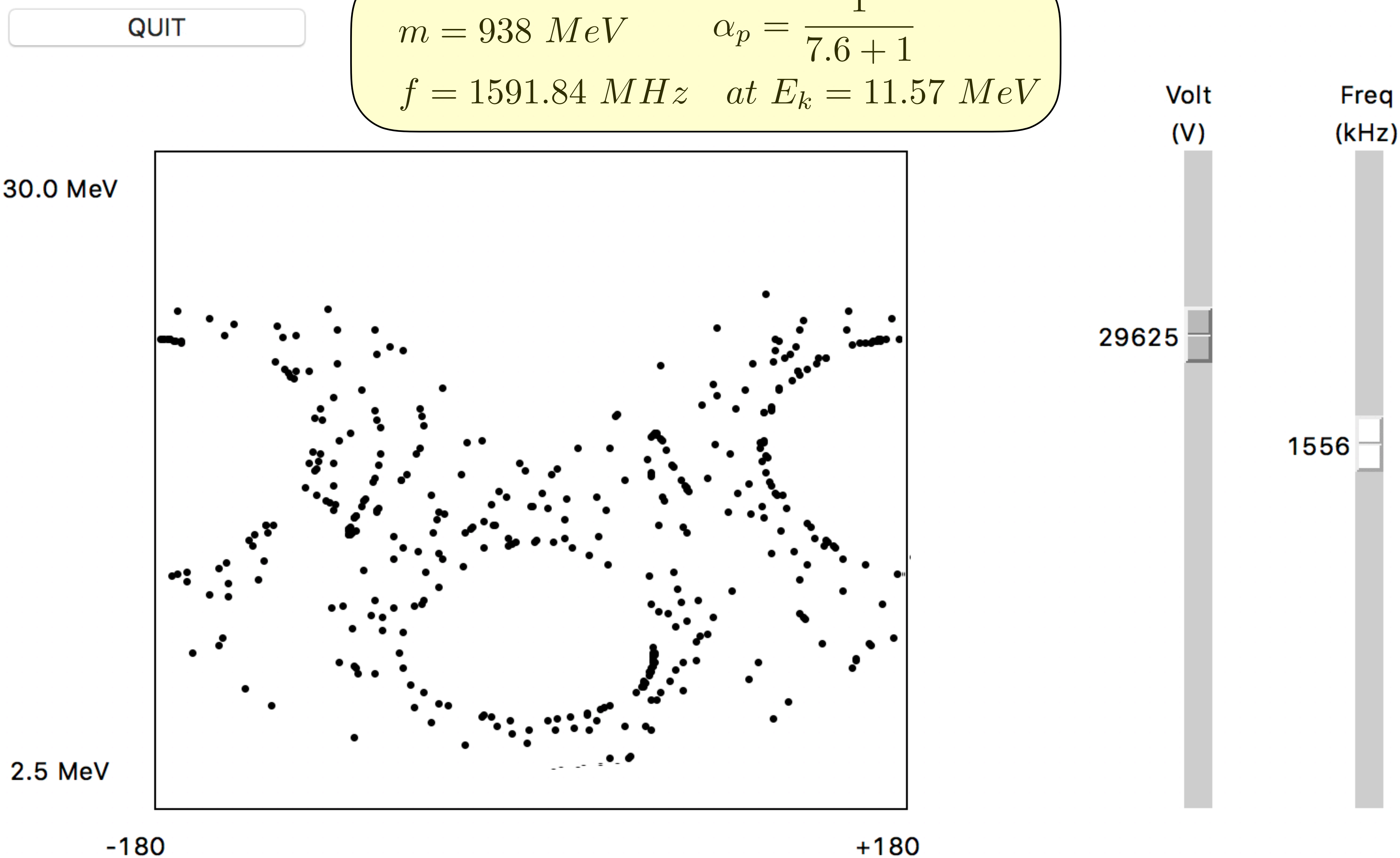


Serpentine acceleration

$$m = 938\ MeV \qquad \alpha_p = \frac{1}{7.6+1}$$

$$f = 1591.84\ MHz \quad at\ E_k = 11.57\ MeV$$



H. Okita, "Beam study of MERIT FFAG", in this workshop.

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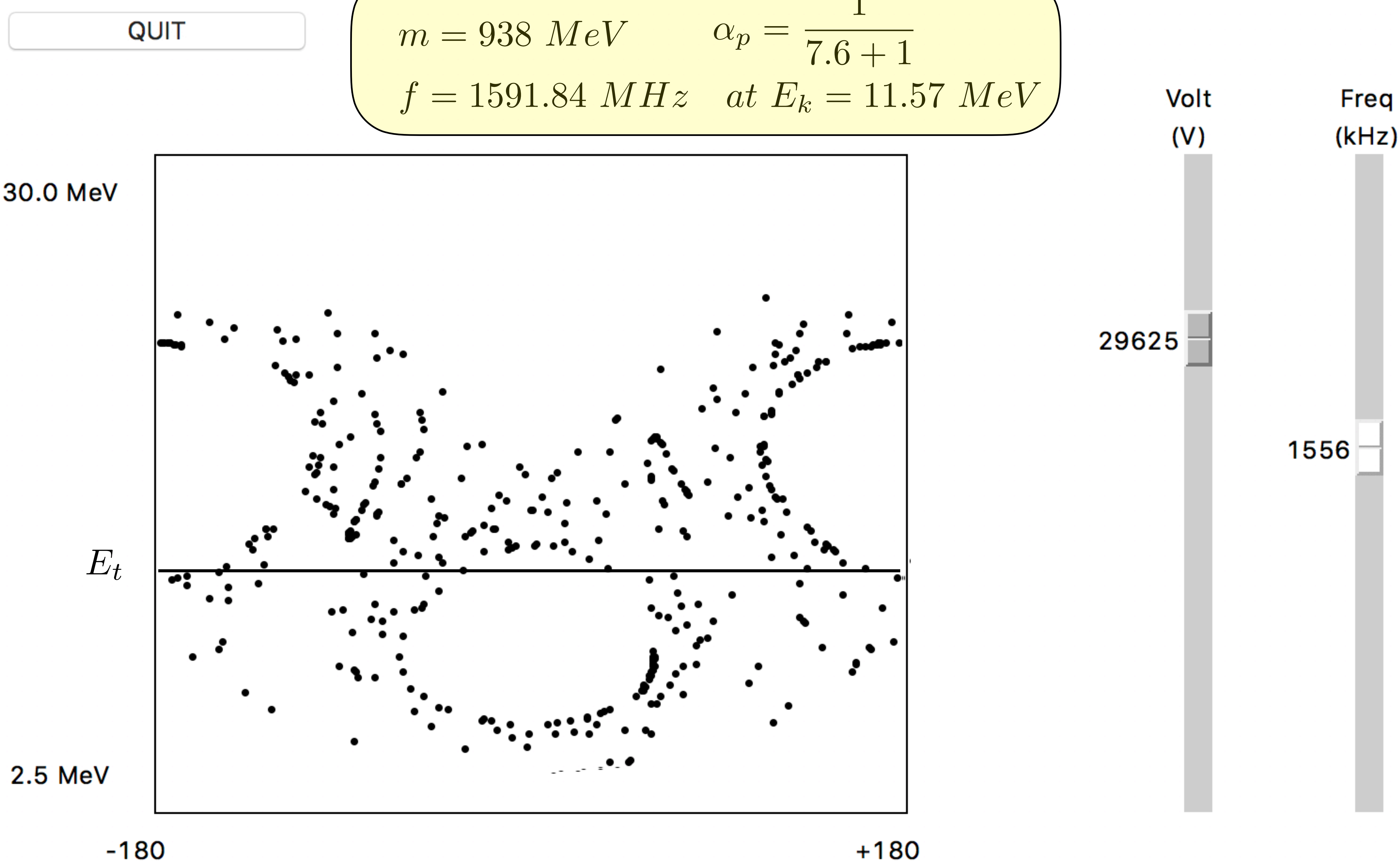

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Serpentine acceleration

QUIT

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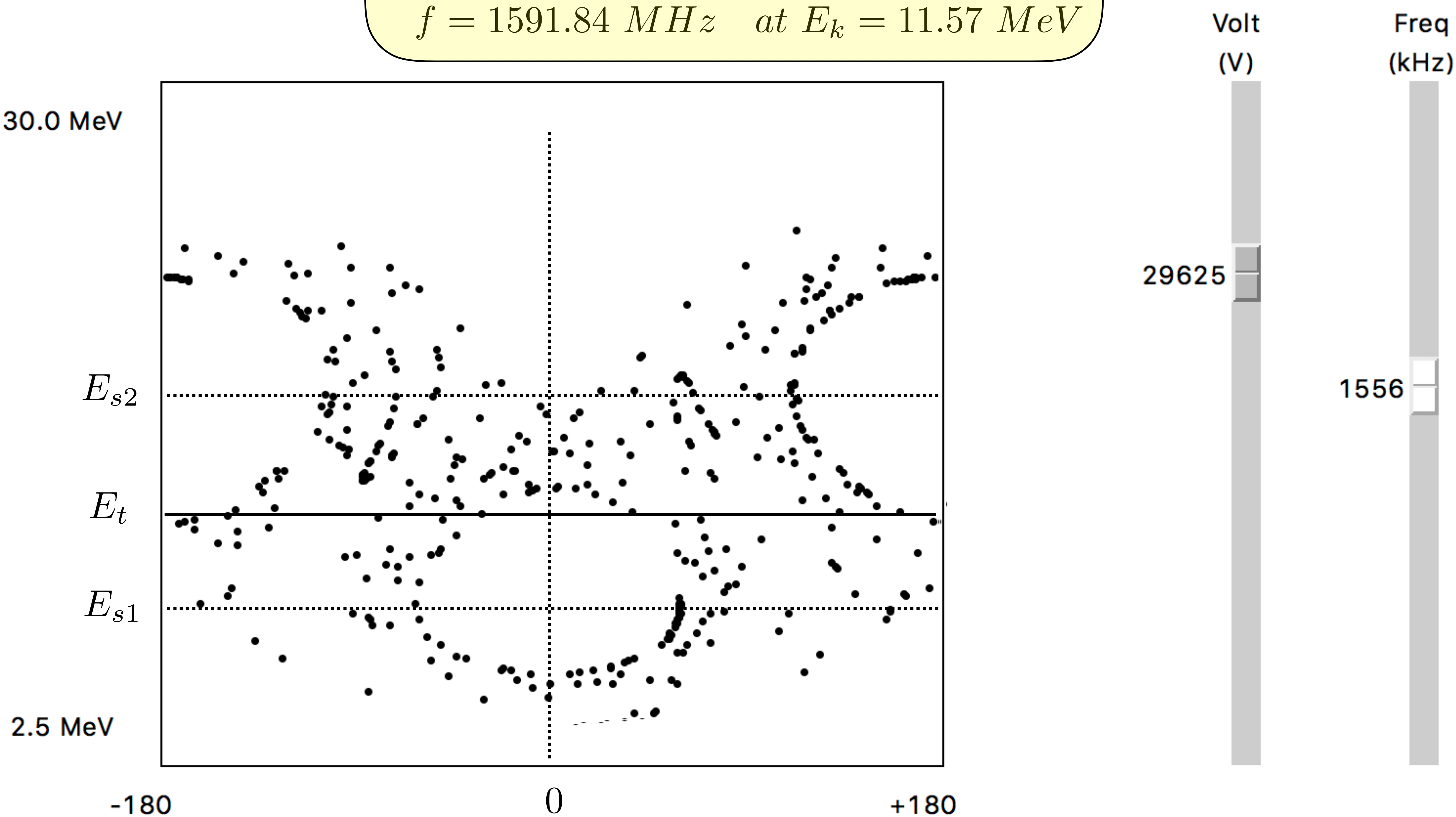

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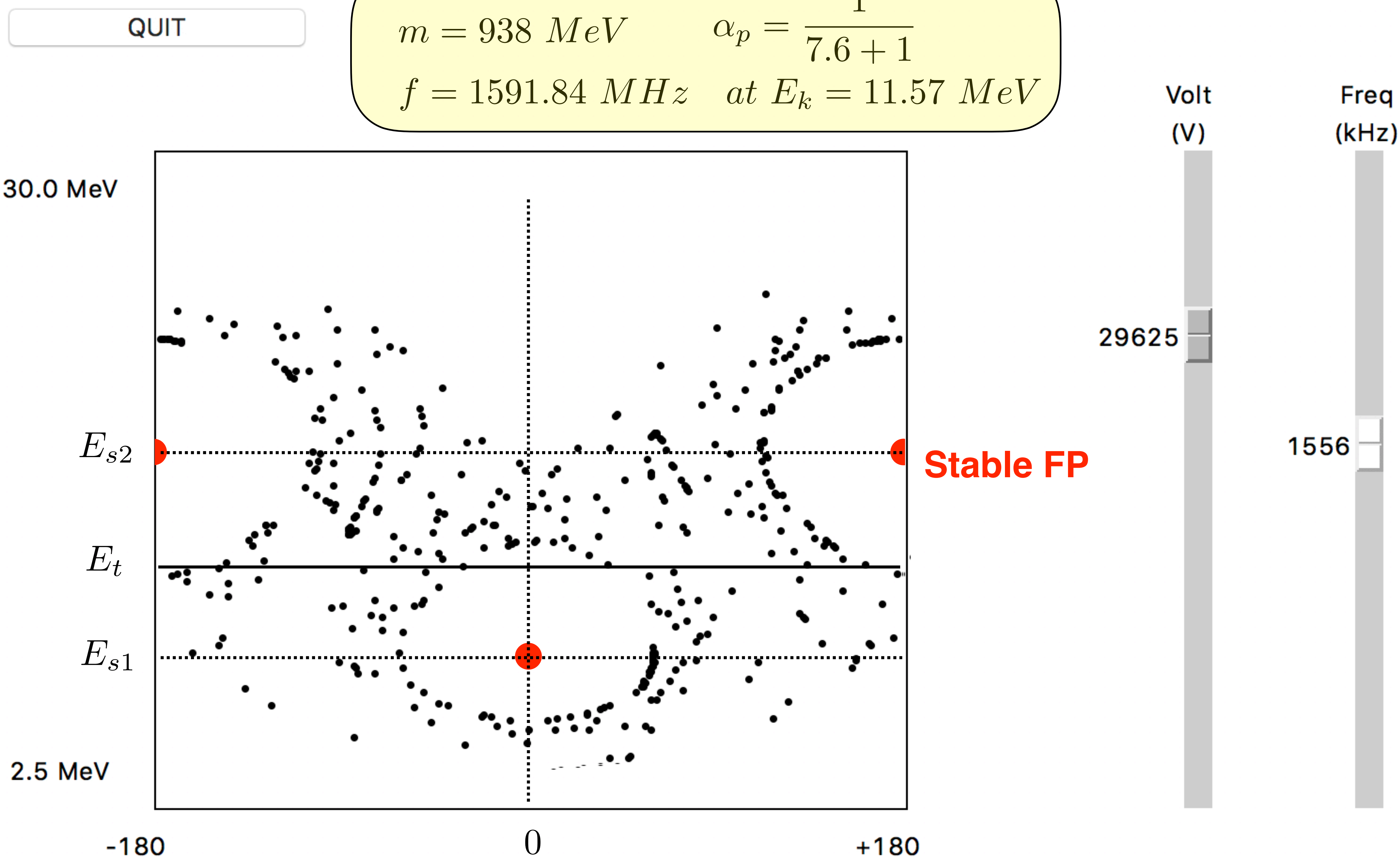

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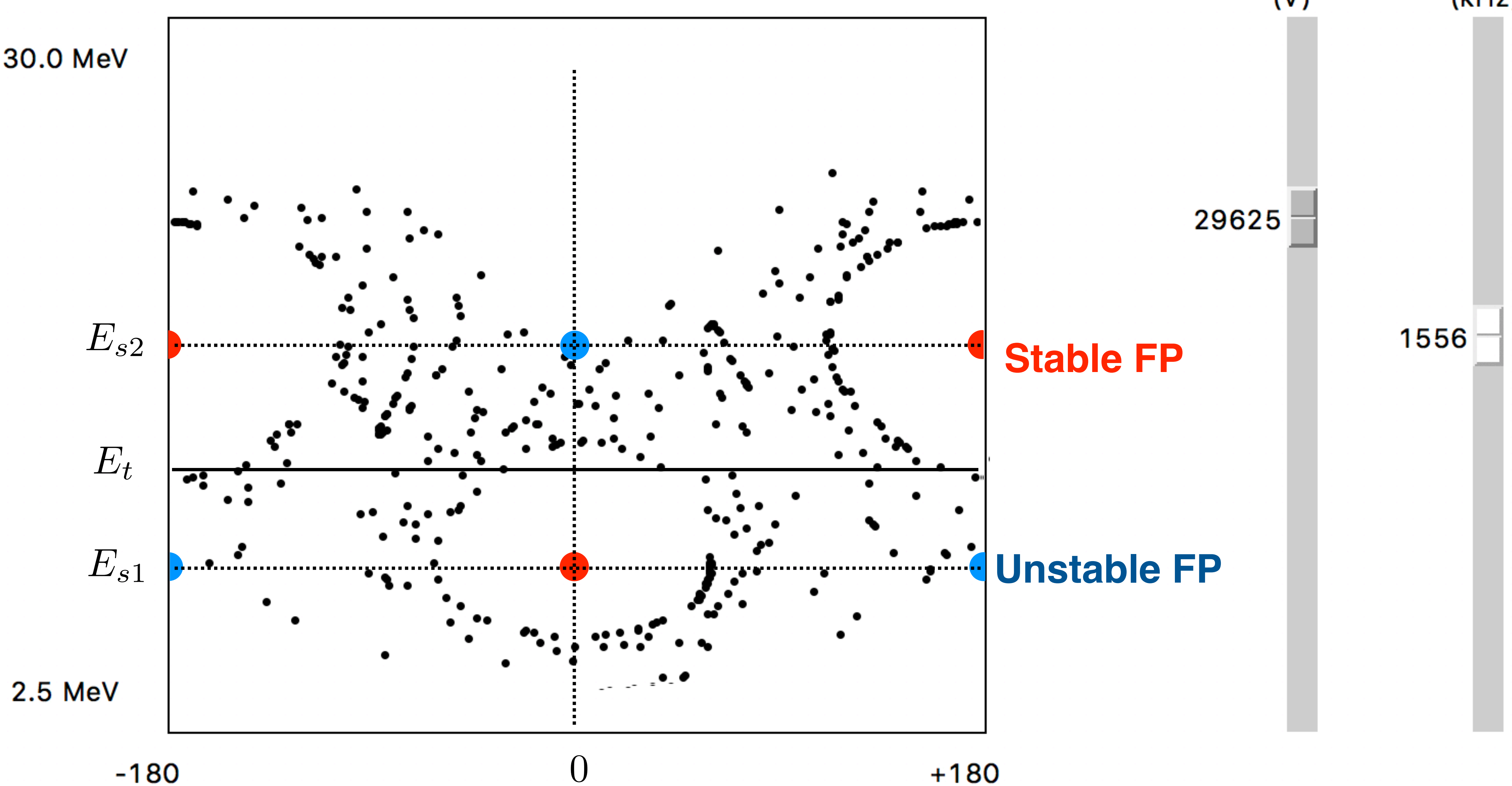

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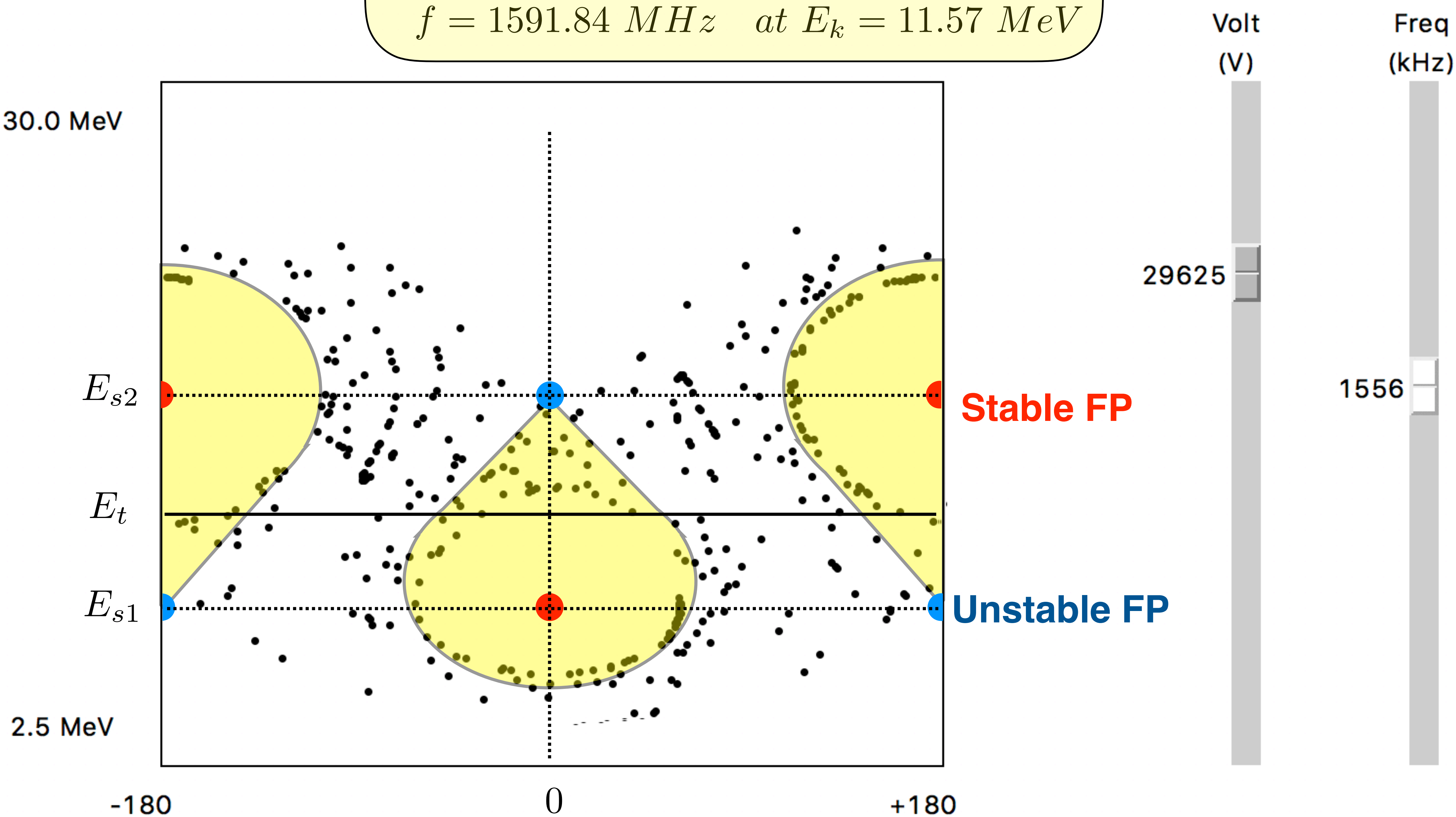

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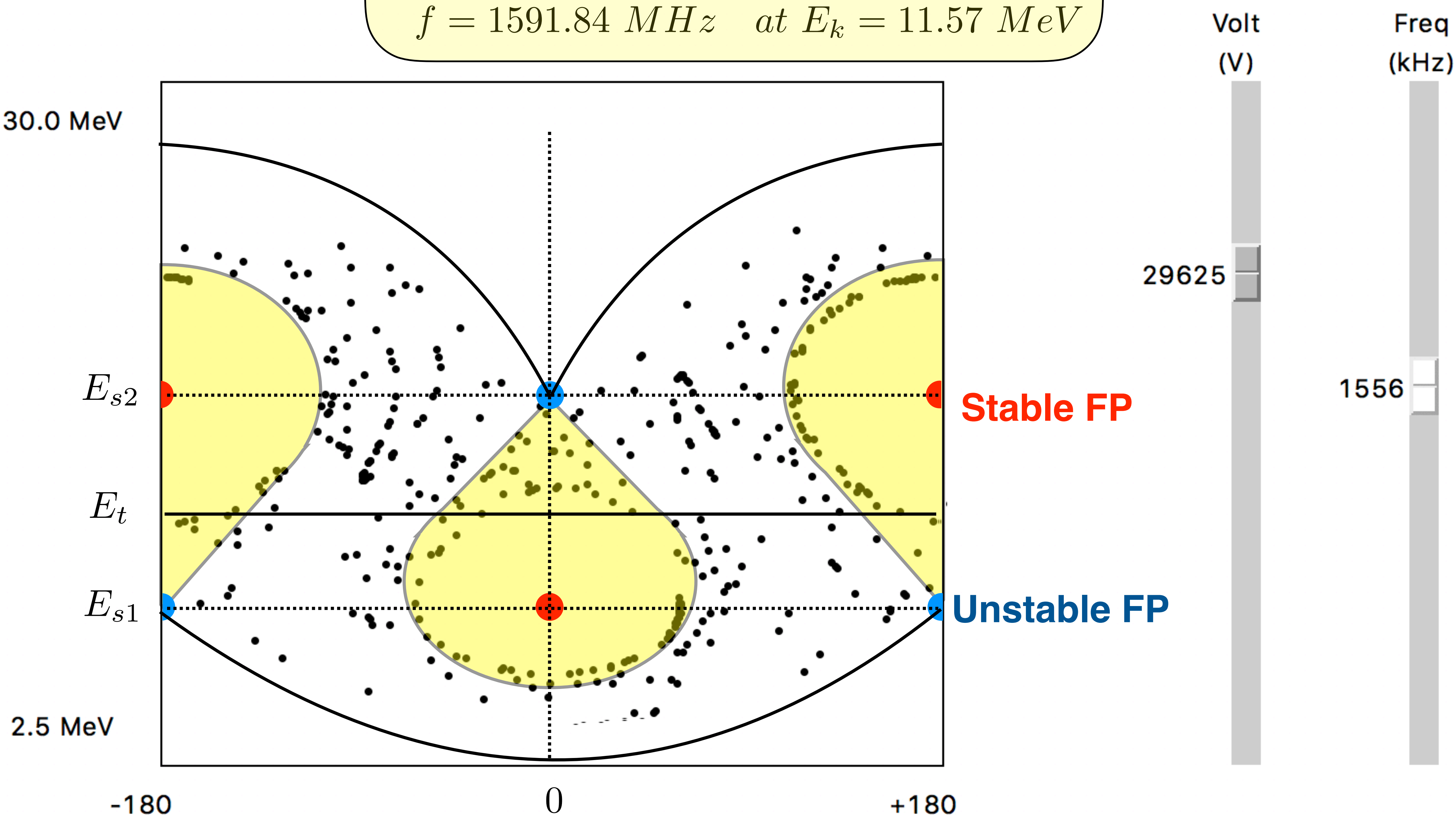

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Harmonic number jump acceleration

Energy

Frequency

Azimuthal position

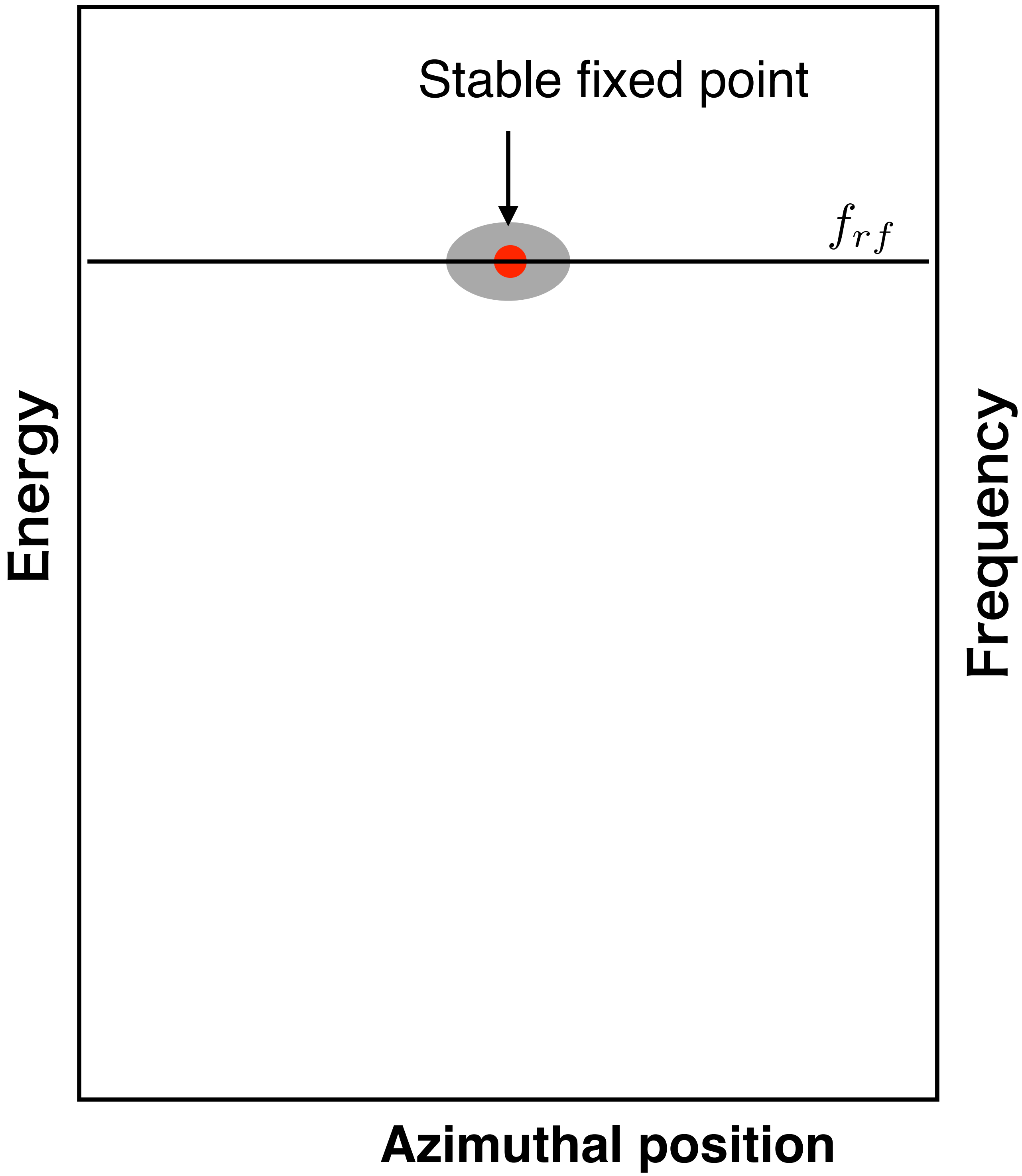
Let us recall

$$f_{rf} = h \cdot f(E_s)$$

A. G. Ruggiero, "Rf acceleration with harmonic number jump", Phys. Rev. ST AB 9(10), 2006.

Y. Mori, "Harmotron", in this workshop.

Harmonic number jump acceleration



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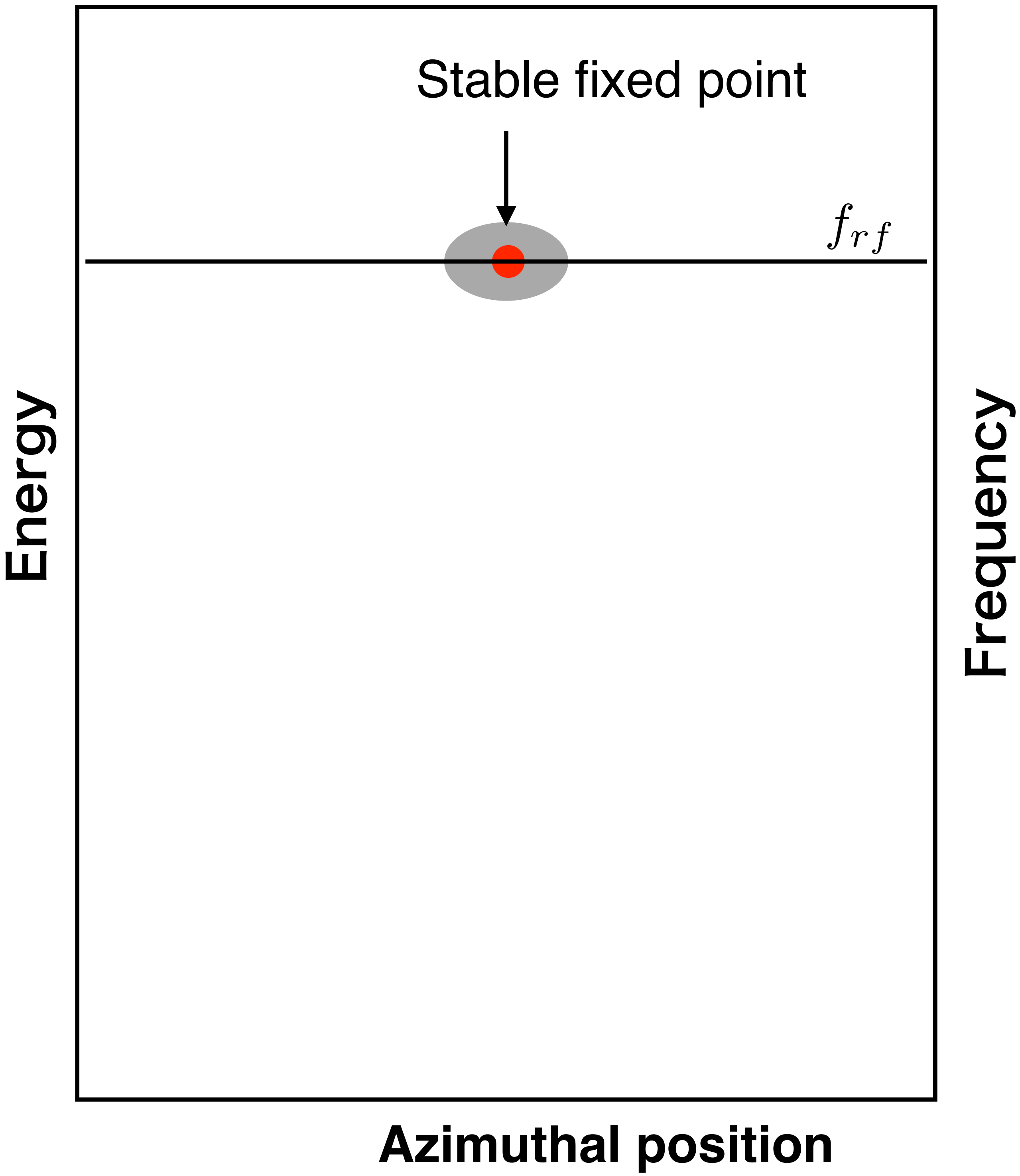

Let us recall

$$f_{rf} = h \cdot f(E_s)$$

Synchronous energy E_s is not unique, because h can be any integer.

There are many stable fixed points, corresponding to h=1,2,3,4,…

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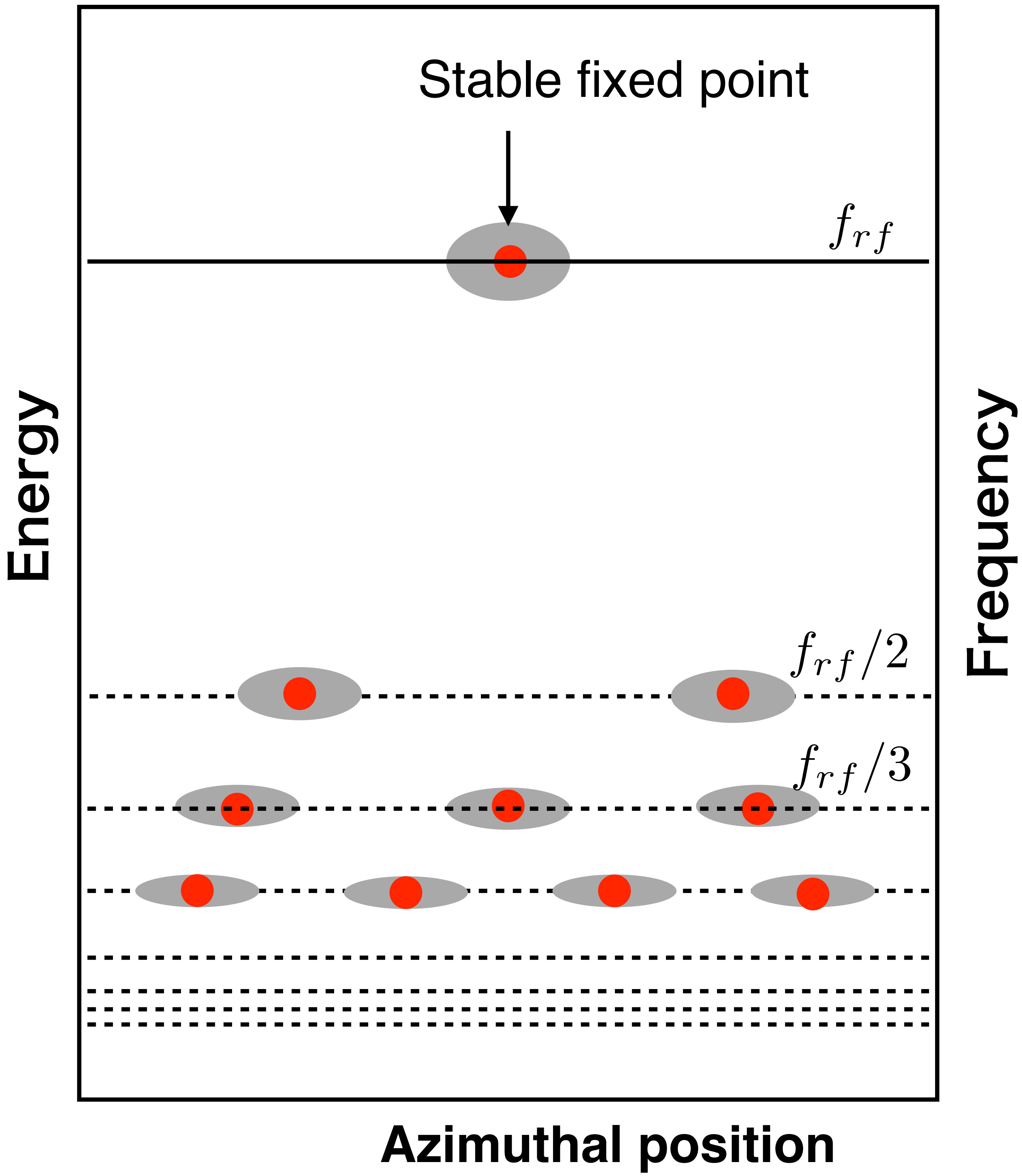

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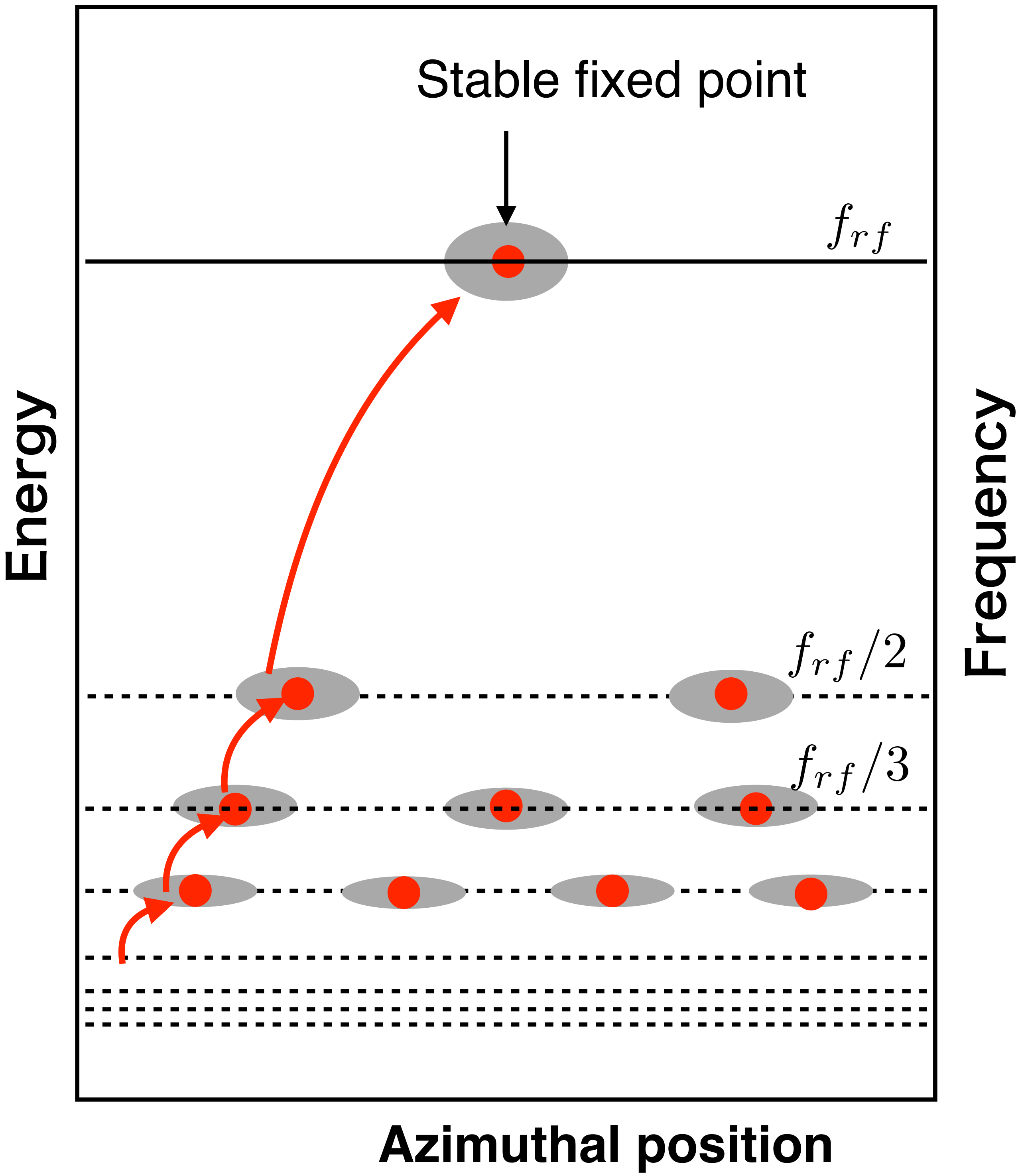

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Synchronous energy E_s is not unique, because h can be any integer.

There are many stable fixed points, corresponding to h=1,2,3,4,…

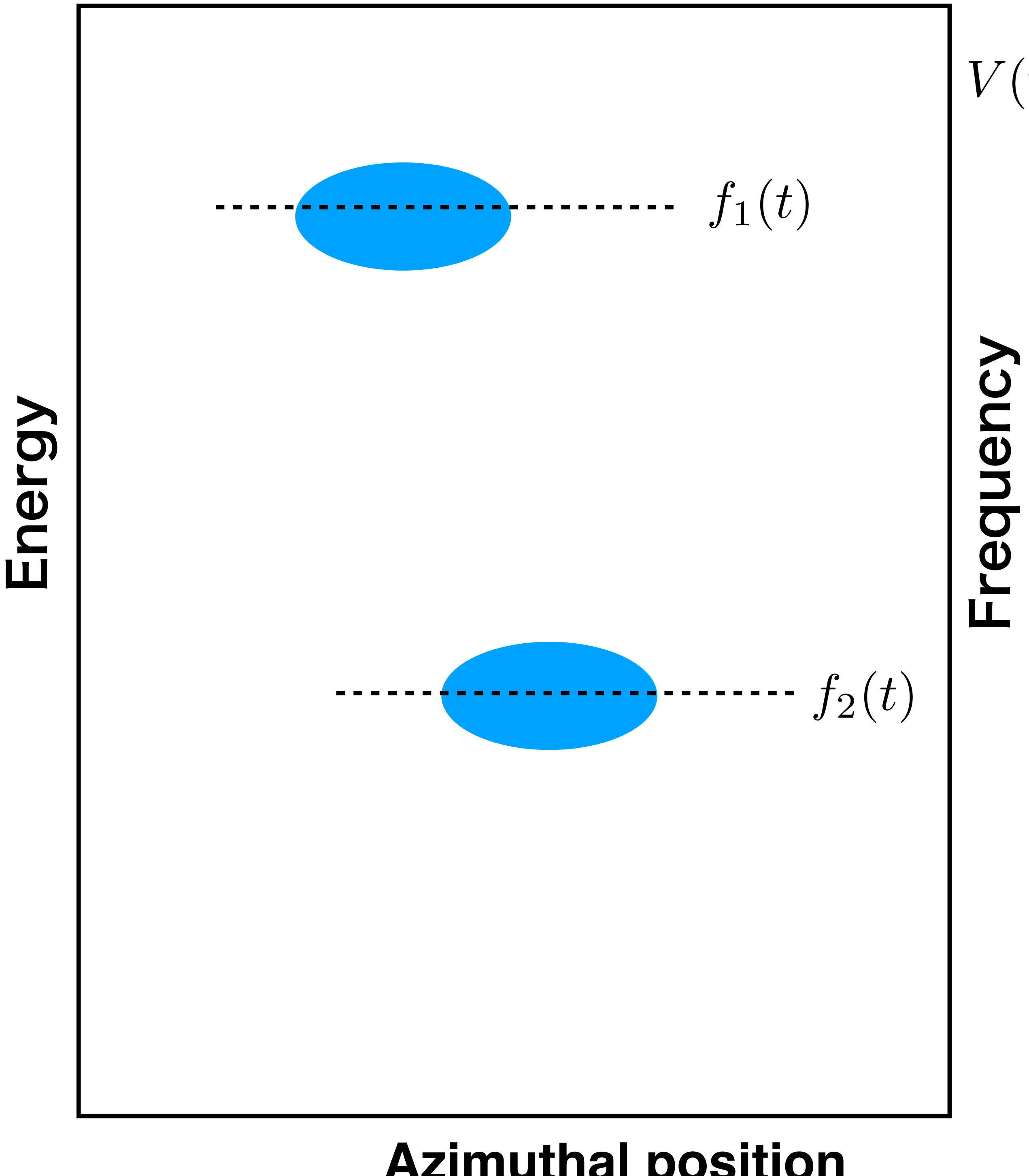
Acceleration across different h's is possible,

if **voltage is high enough** and **slippage is tuned well**.

A. G. Ruggiero, "Rf acceleration with harmonic number jump", Phys. Rev. ST AB 9(10), 2006.
Y. Mori, "Harmotron", in this workshop.

Multi-fish

Mixing two (or more) rf component with different frequencies,



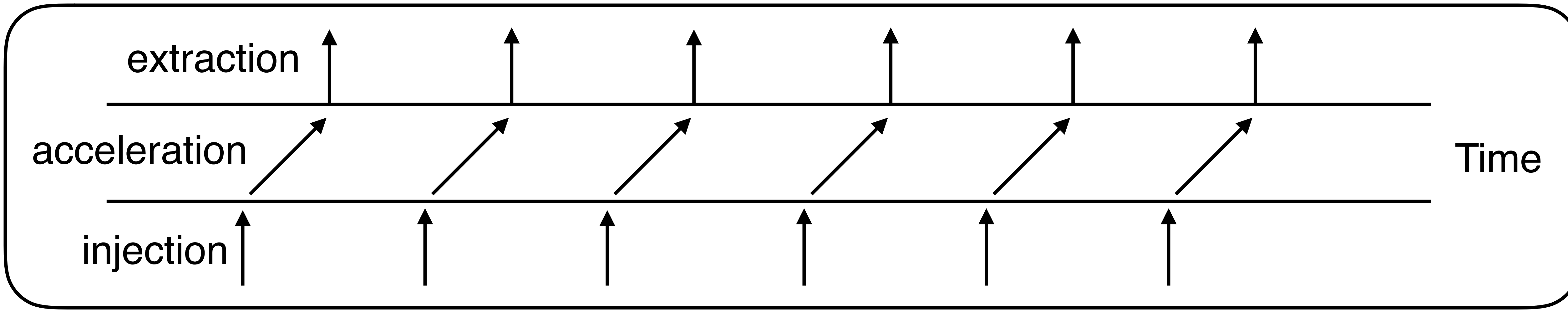
$$V(t) = V_1 \sin\left(\int 2\pi f_1(t)\, dt\right) + V_2 \sin\left(\int 2\pi f_2(t)\, dt\right)$$

Repetition rate becomes twice (or higher).

A new beam is injected during acceleration.

Frequencies must be separated enough each other, otherwise two rf interferes.

Rf stacking at extraction energy



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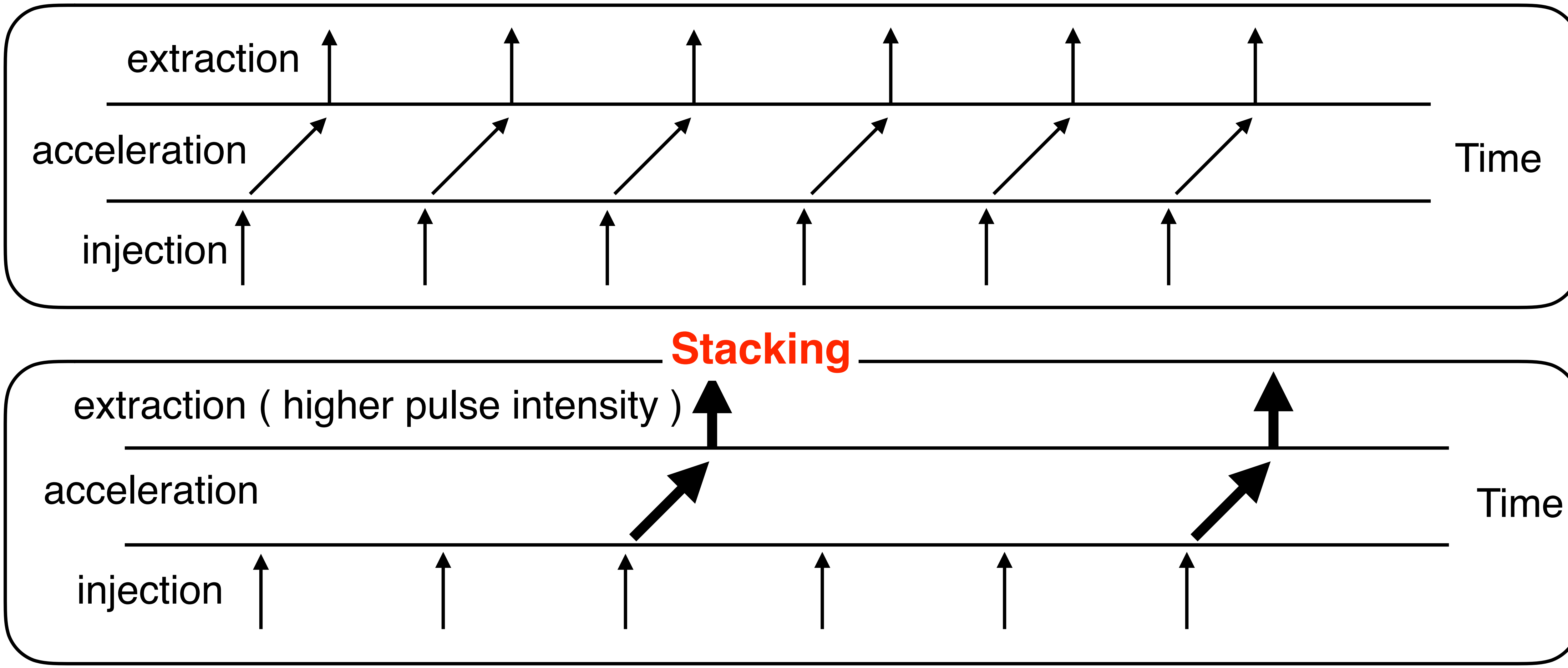

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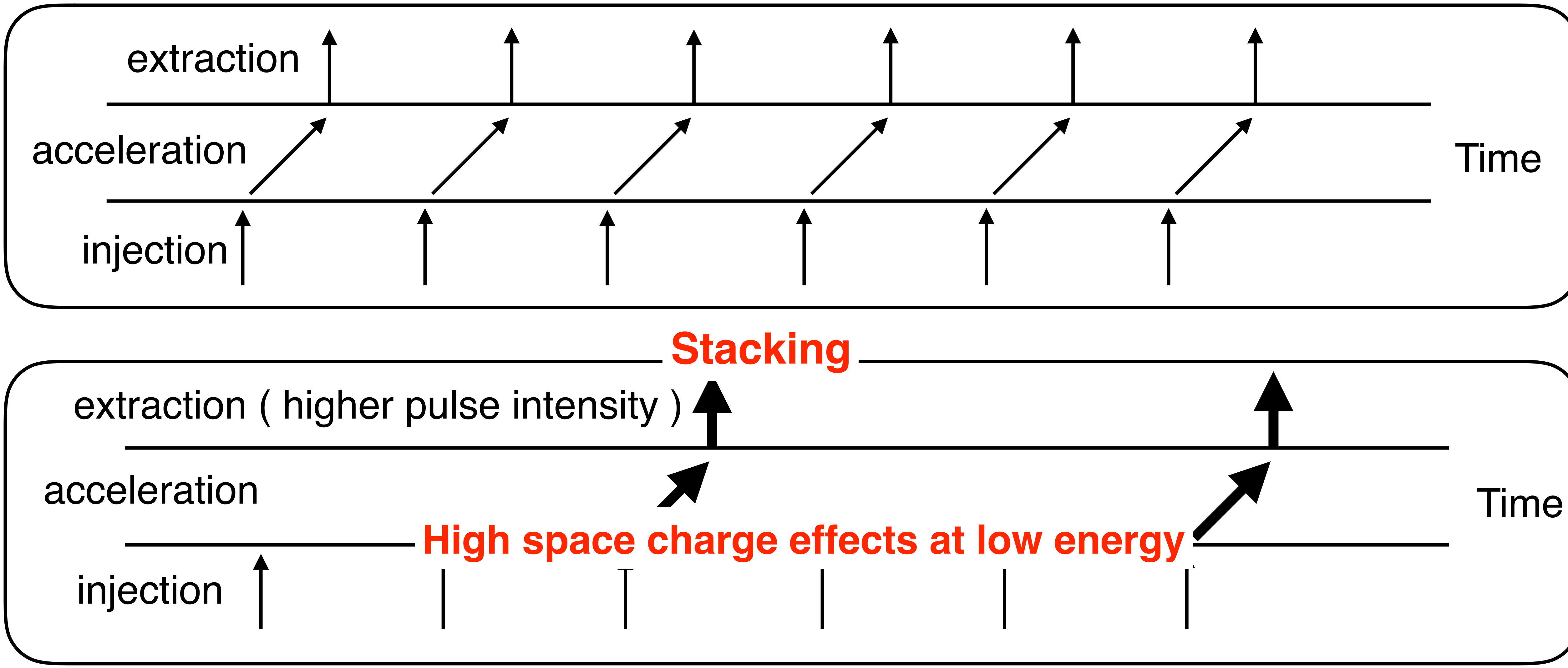

Tom UESUGI, FFA SCHOOL, Osaka, Sep, 2018

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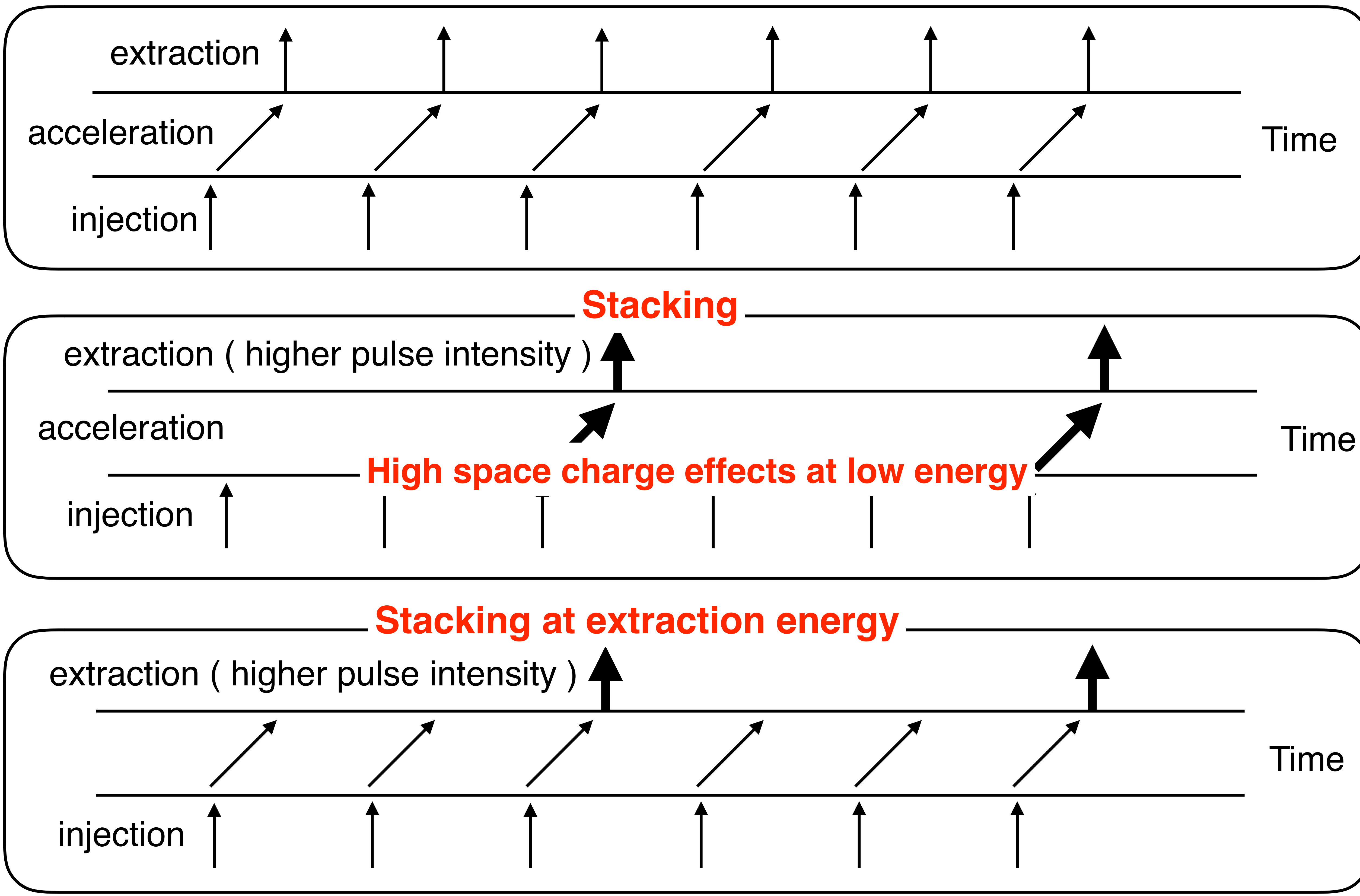

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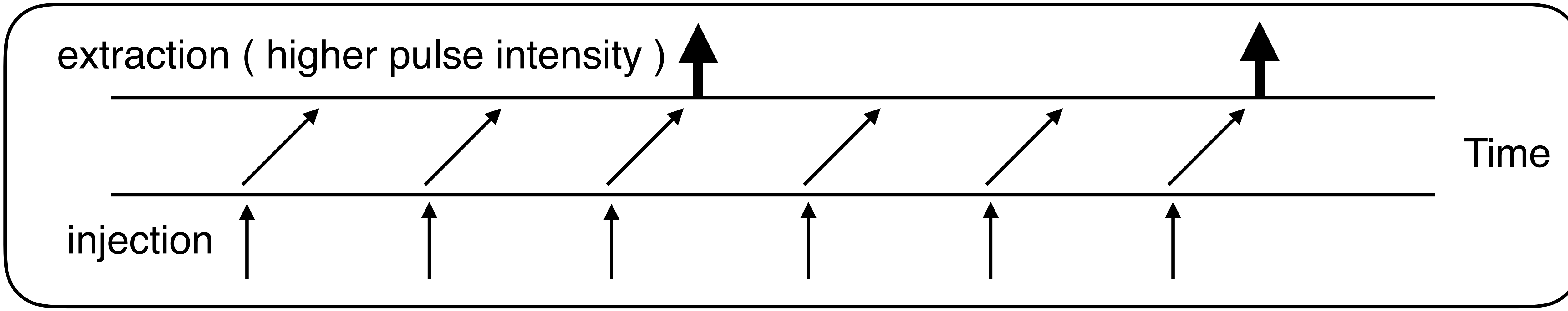
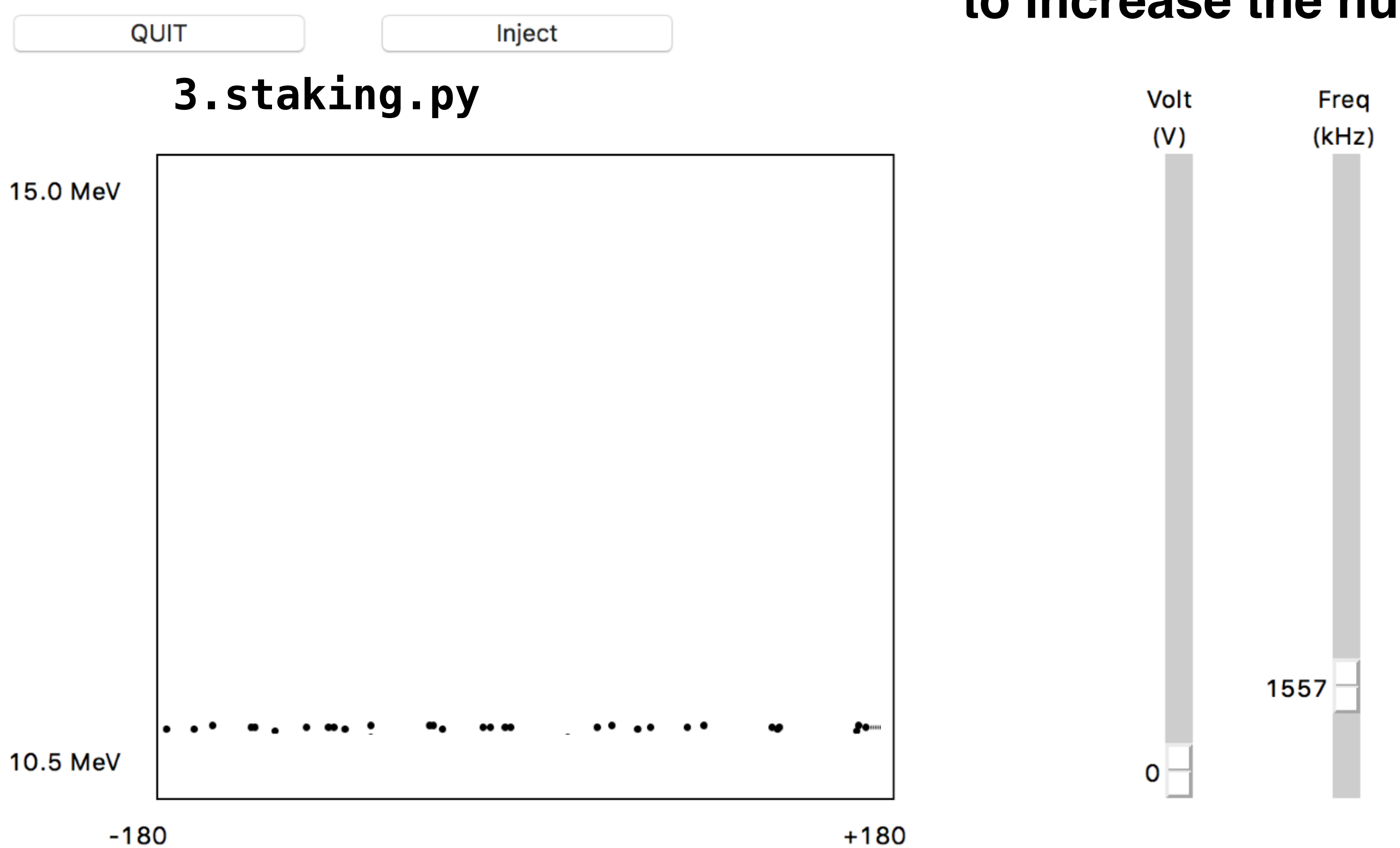


Beam is extracted after two or more times of injection.
to increase the number of particle per pulse.



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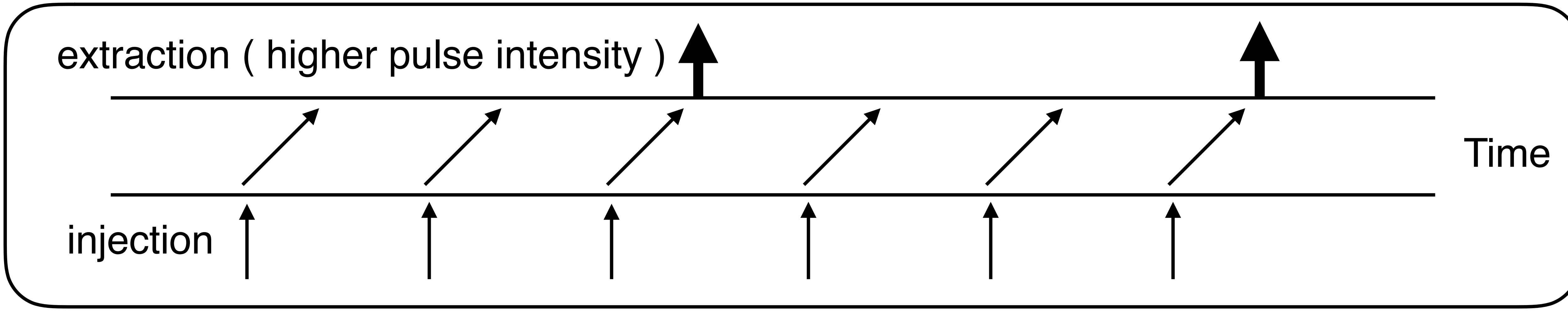

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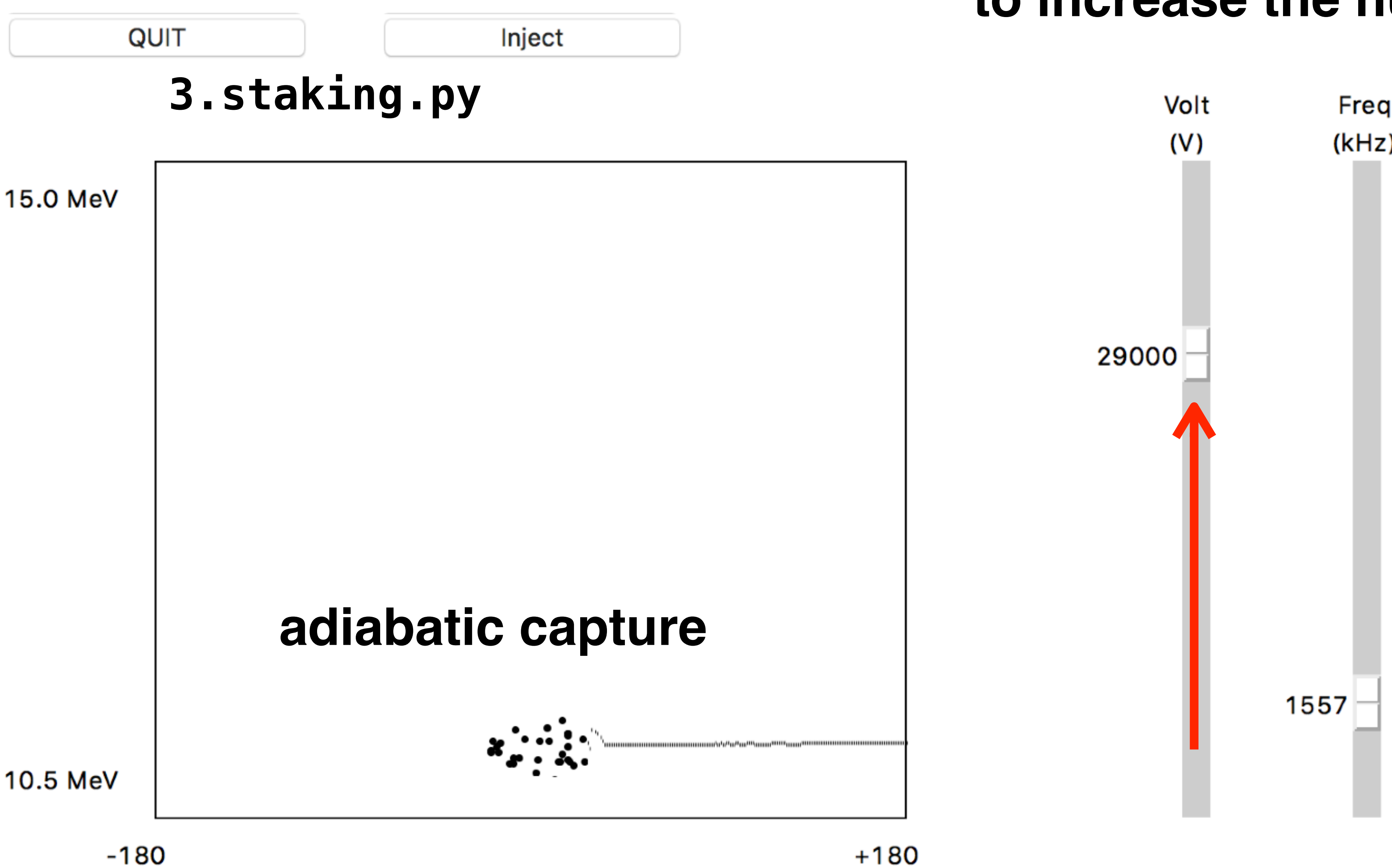

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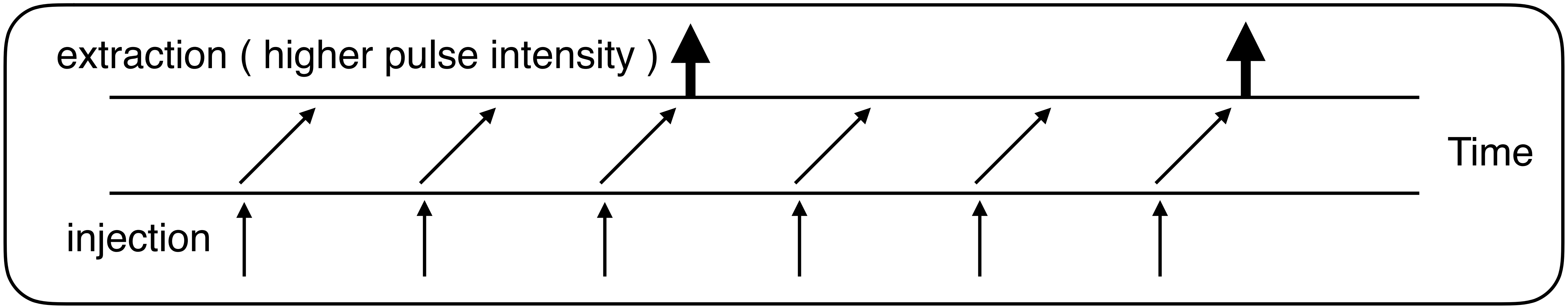

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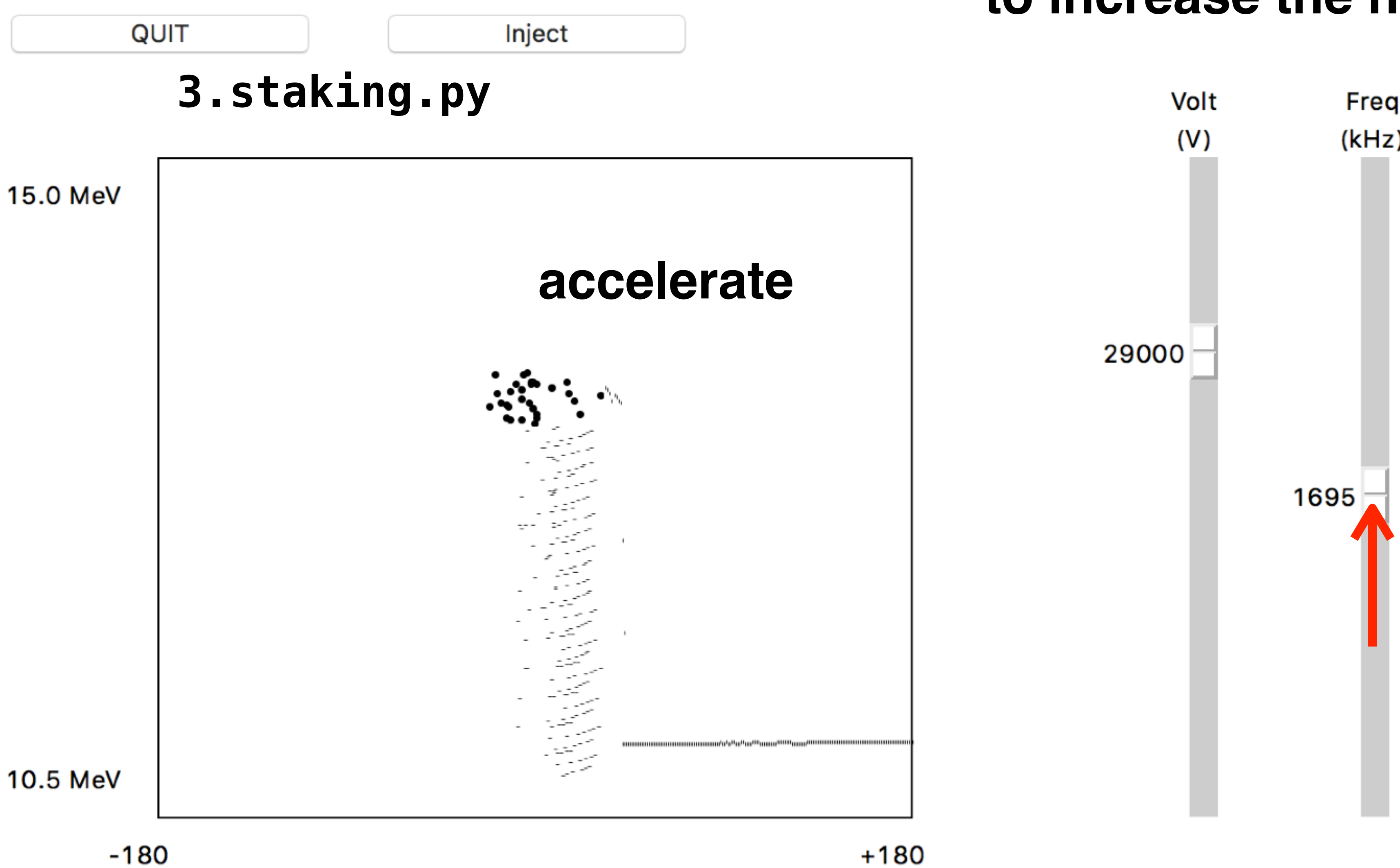

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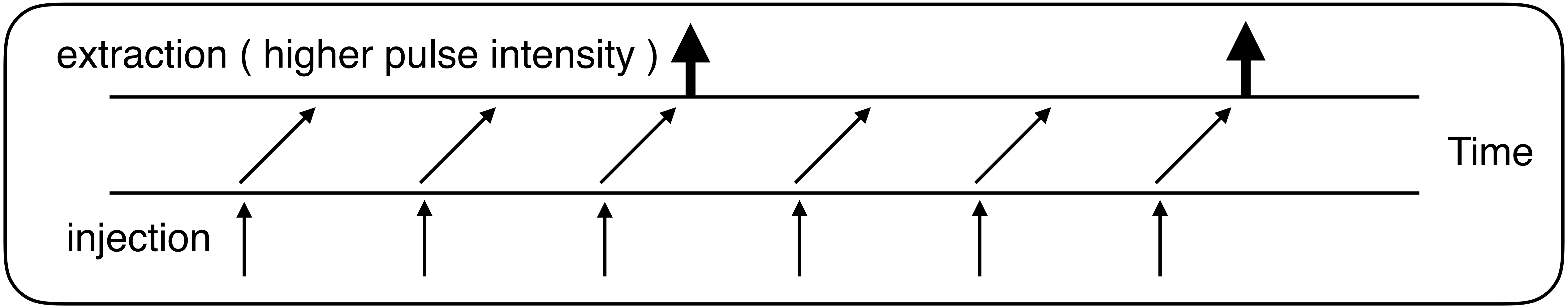

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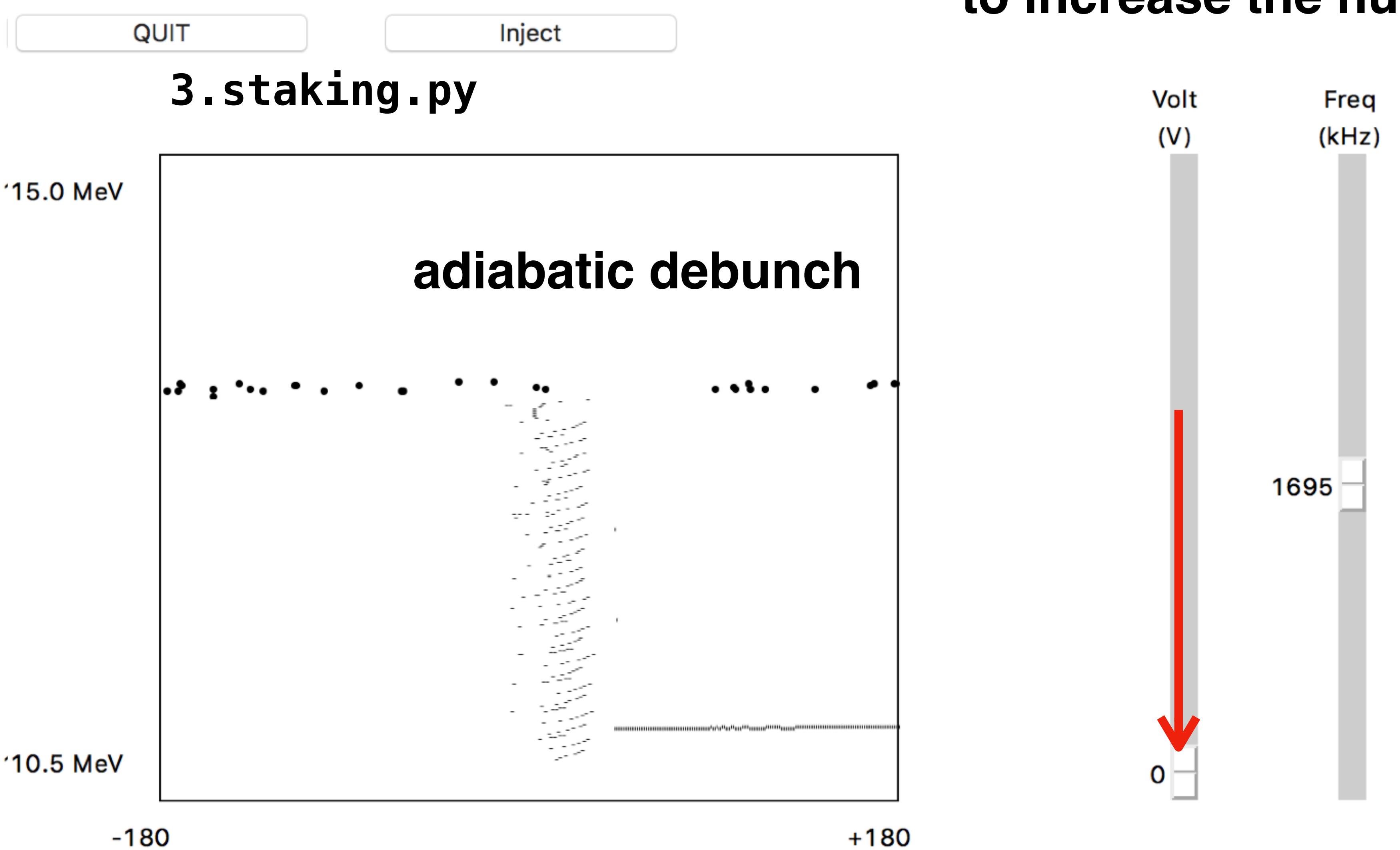

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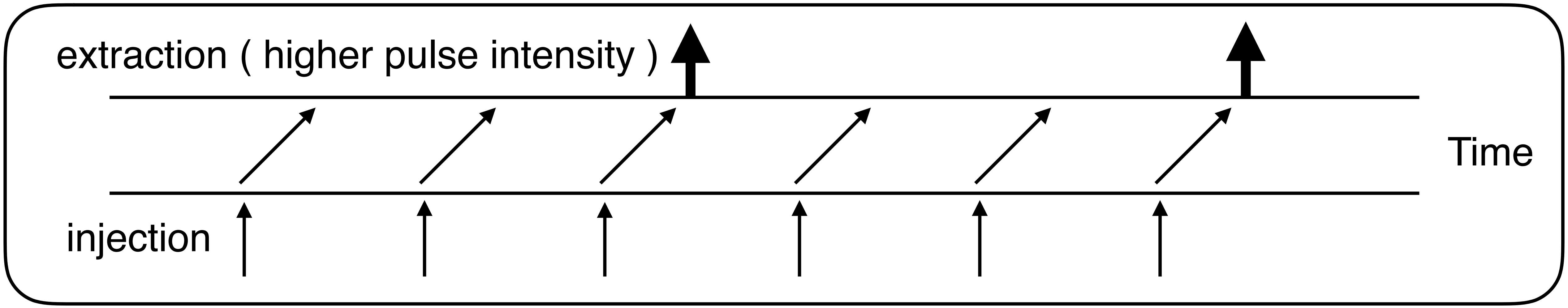

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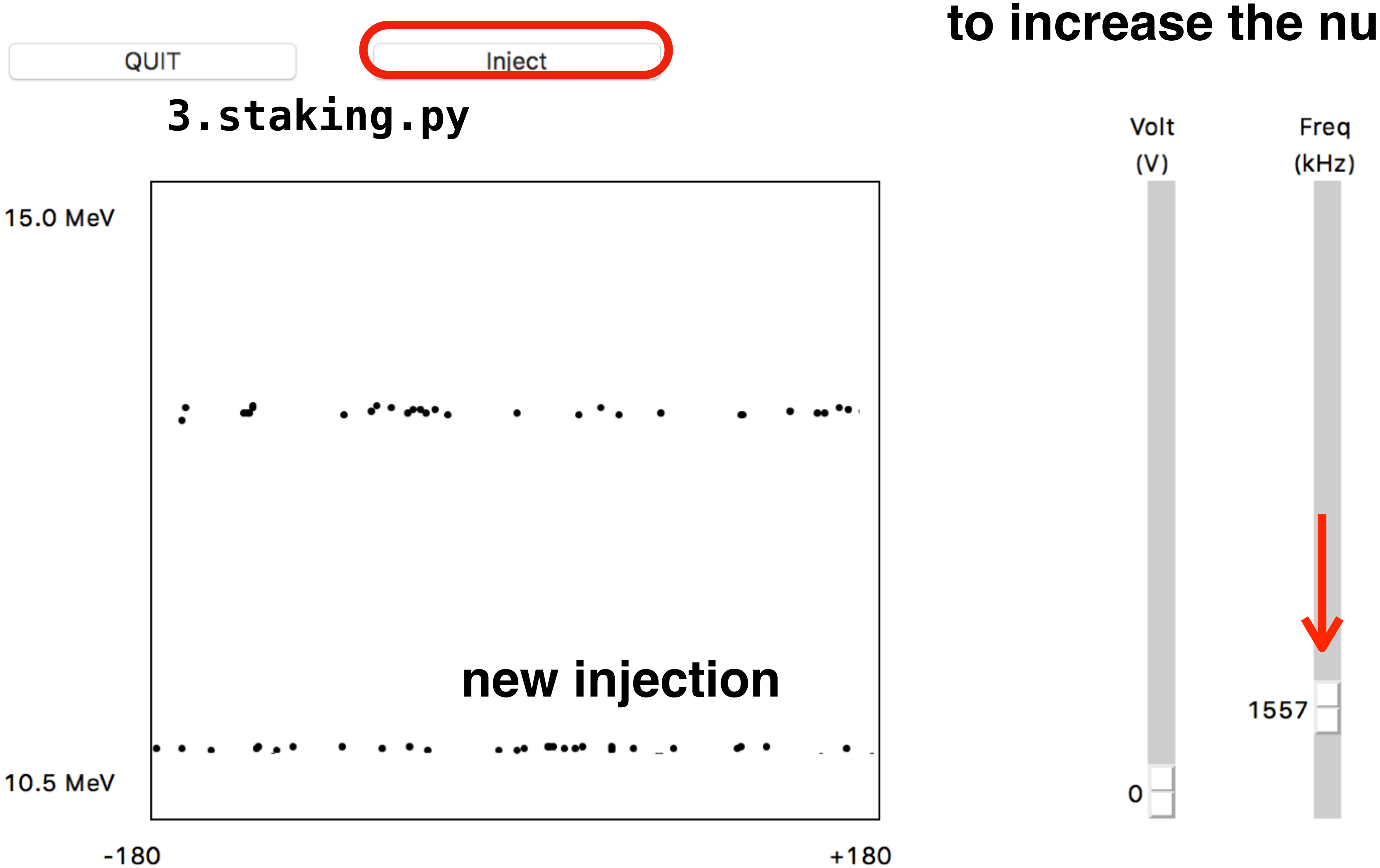

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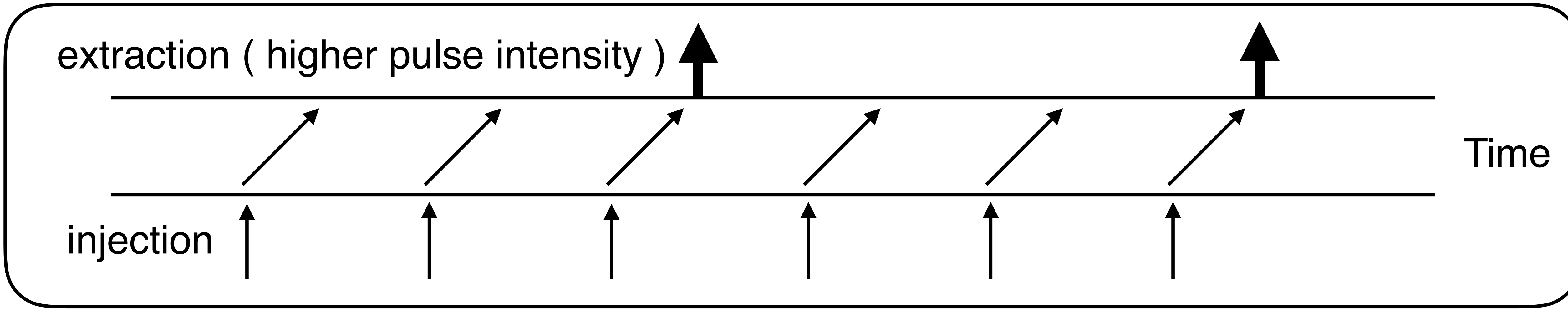

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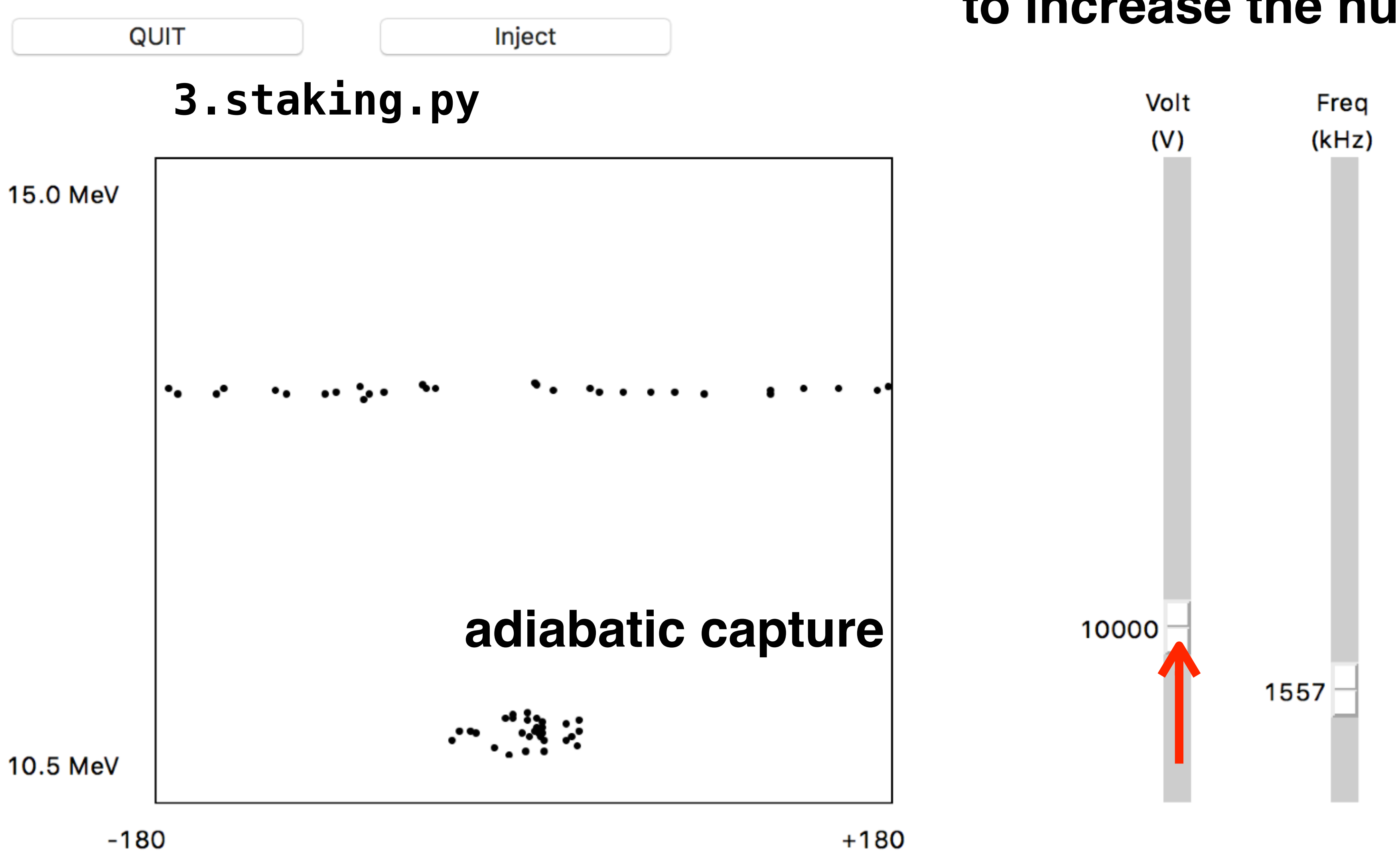

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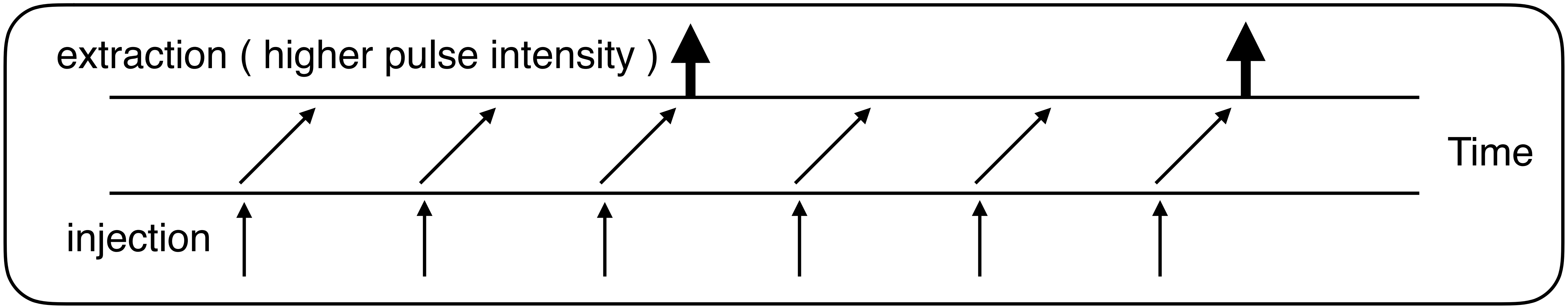

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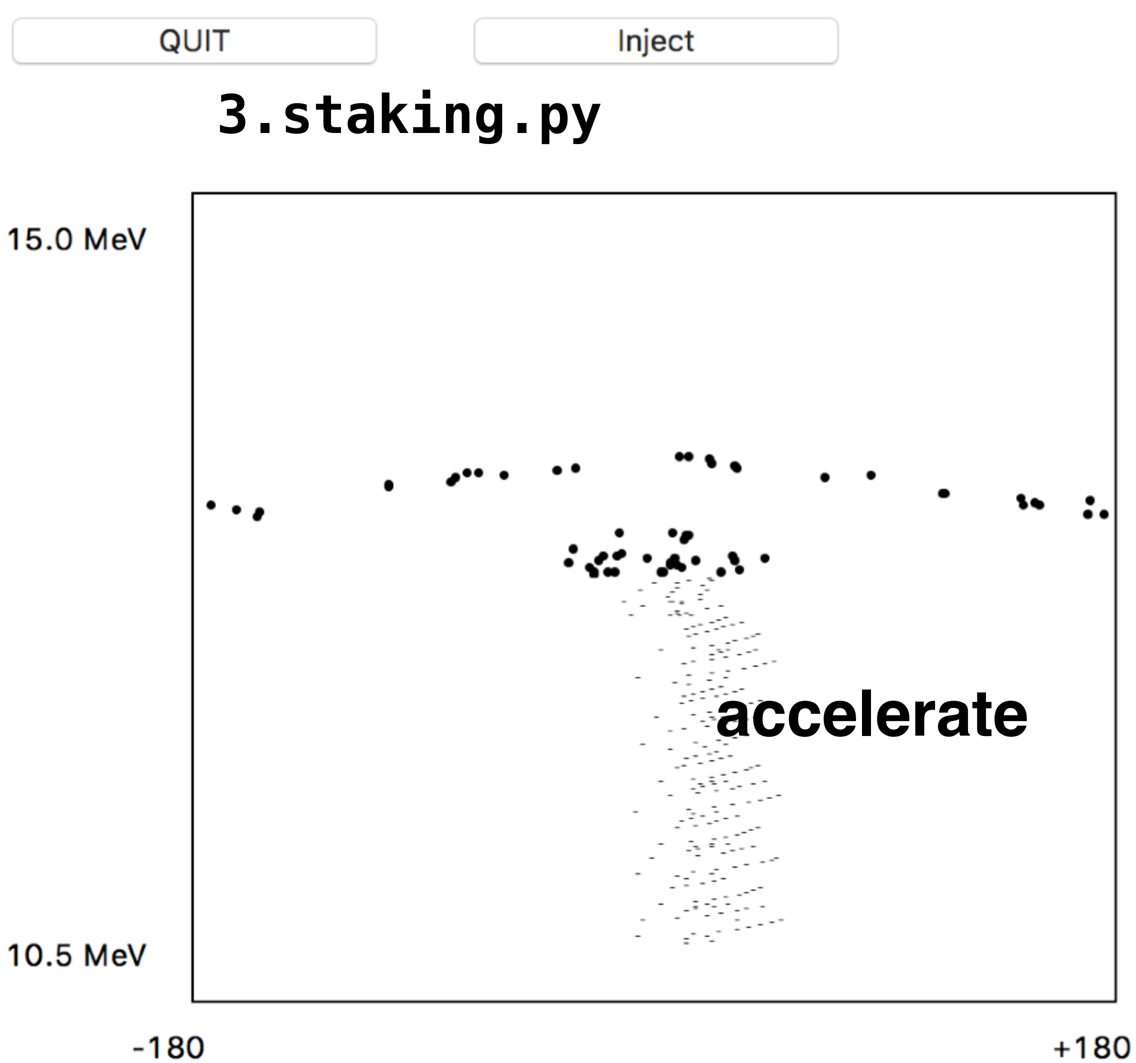

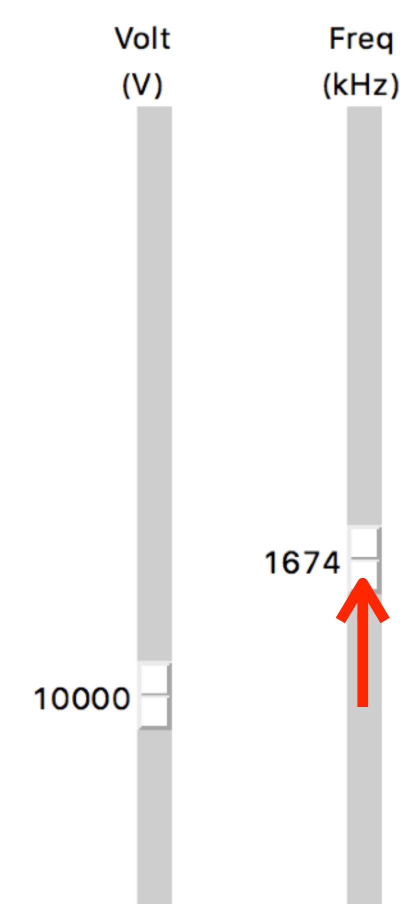

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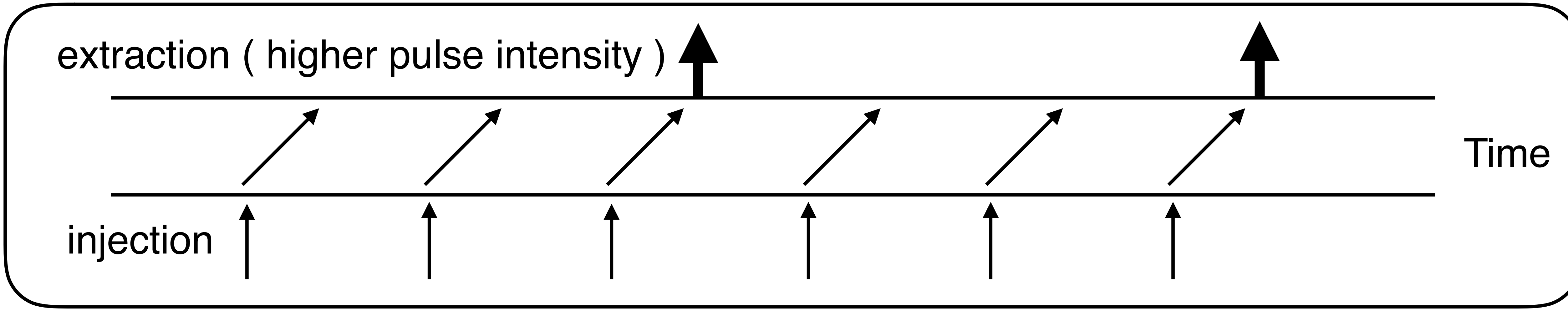

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